Building an FFAG or a Cyclotron around its Orbits

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Fixed field lattice with mid-plane symmetry

Ingredients:

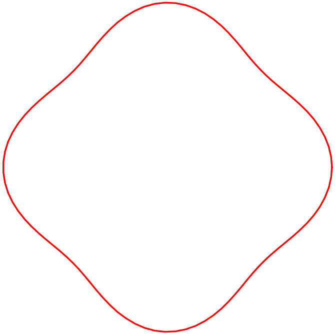
- Continuum of closed orbits.
- Relation between length of the orbit and energy.

To obtain:

- Field distribution.
- Tunes without tracking particles (?).

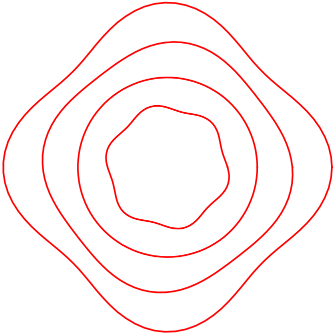
One closed orbits

$$R_{CO}(\theta) = r_0\left(1 + \sum A_n \cos(n\theta + \varphi_n)\right)$$



Continuum of closed orbits

$$R_{CO}(\theta, a) = a \cdot r_0 \left(1 + \sum A_n(a) \cos(n\theta + \varphi_n(a))\right)$$



Continuum of closed orbits

Here we assume that $R_{CO}(\theta, a)$ is bijective.

$P(a)$

Length of each closed orbit:

$$L(a) = \int_0^{2\pi} \sqrt{R_{CO}(\theta,a)^2 + R'_{CO}(\theta,a)^2}\,\mathrm{d}\theta$$

Your choice: $L(E)$:

- Isochronous condition: $L \propto \beta$,
- 'Scaling FFAG like': $L \propto P^{\frac{1}{k+1}}$,
- ...

Field distribution

$$\rho = \frac{U'' + U}{\left(1 + \frac{U'^2}{U^2}\right)^{3/2}}, \qquad \text{with } U = \frac{1}{R_{CO}}$$

Field distribution

and $B = \dfrac{B\rho}{\rho}$

Example

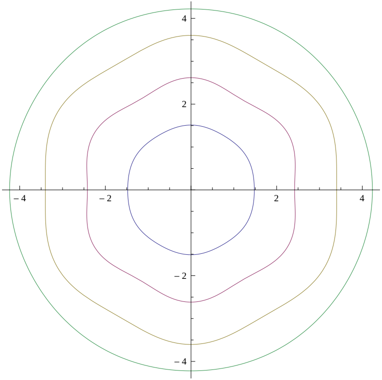


Fig. 1: Closed orbits of 10, 30, 60 and 90 MeV protons. 'TRIUMF cyclotron like' basic parameters ($B_{conv} = 0.303$ T, 6 sectors)

Example

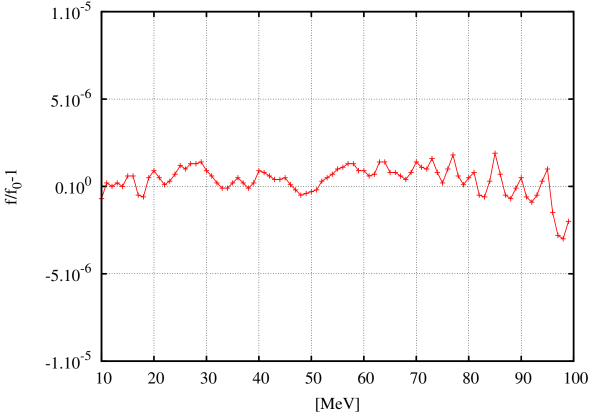


Fig. 2: Revolution frequency calculated by CYCLOPS

Example

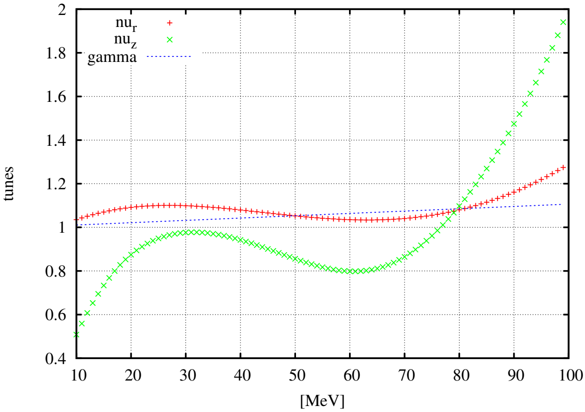


Fig. 3: Tunes calculated by CYCLOPS

Example

$$\nu_x^2 = \langle \mu^2(1-n) \rangle_{\text{Av}} + \langle \{ \mu^2(1-n) \}_1^2 \rangle_{\text{Av}}, \qquad (6.3)$$

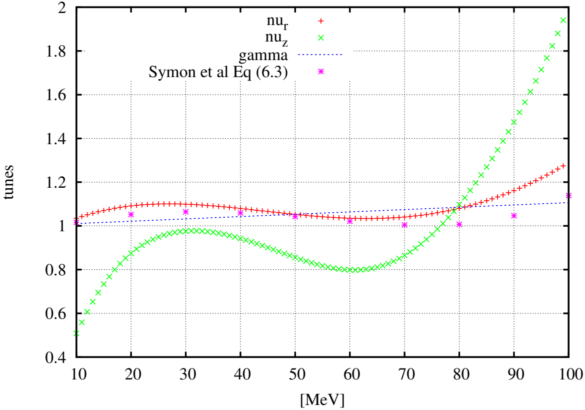
$$\nu_z = \langle \mu^2 n \rangle_{\text{Av}} + \langle \{ \mu^2 n \}_1^2 \rangle_{\text{Av}}. \qquad (6.4)$$

$$\nu_x^2 = k+1+\frac{(k+1)^2 f^2}{N^2} \langle g_1^2 \rangle_{\text{Av}}, \qquad (6.7)$$

$$\nu_z^2 = -k+\frac{f^2}{2}+\frac{(k-1)^2 f^2}{N^2} \langle g_1^2 \rangle_{\text{Av}}, \qquad (6.8)$$

Fig. 4: From Symon *et al.*, Phys. Rev. 103-6, 1956

Example


nu_r
nu_z
gamma
Symon et al Eq (6.3)
tunes
[MeV]
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
10 20 30 40 50 60 70 80 90 100