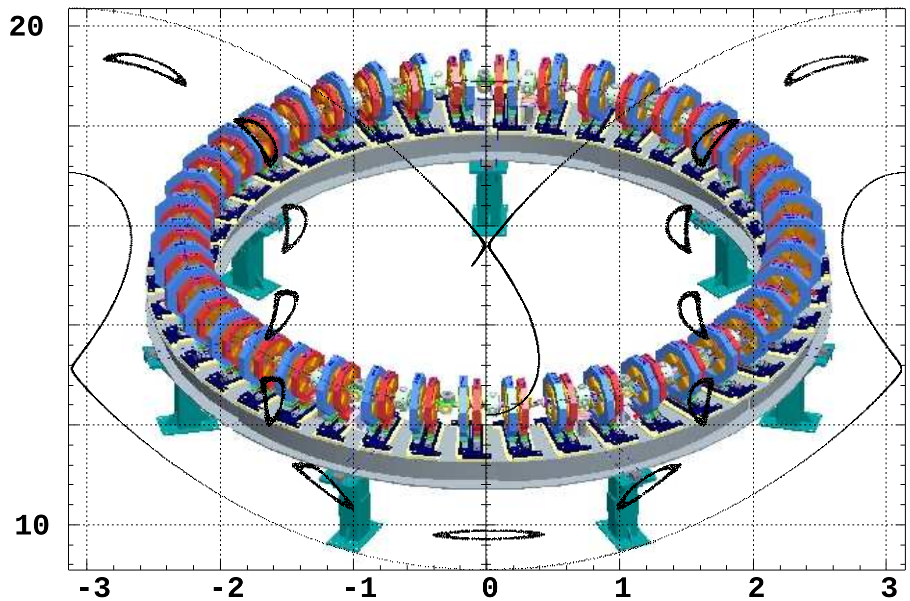
ALL-FFAG 6-D RAY-TRACING CODE



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Contents

1/ **I'll be commenting the version of Zgoubi that I maintained myself, over the years.**

I won't discuss developments done by other groups/people.

2/ **It is available on a development site, together with its "Users' Guide" and its graphic/analysis interface "zpop", and many operational examples**

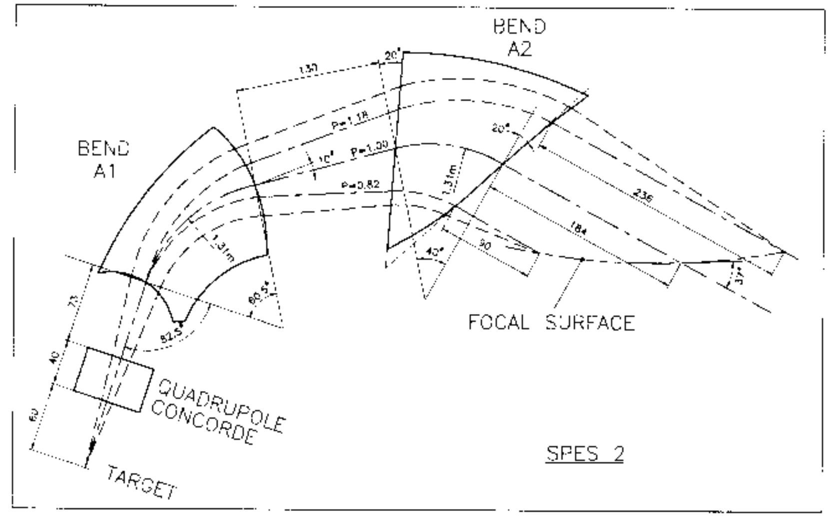
http://sourceforge.net/projects/zgoubi/

A lot of articles and other technical reports can be found on the DOE OSTI site

http://www.osti.gov/bridge/

FFAG'13, TRIUMF, Vancouver, BC, Canada, 21-24 Sept. 2013

... **was written in 1972, at Saclay,**
for SPES2, by J. C. Faivre and D. Garreta



- **The equation of motion in magnets writes**

$$d(m\vec{v}) = q\,\vec{v}\times\vec{b}\,dt$$

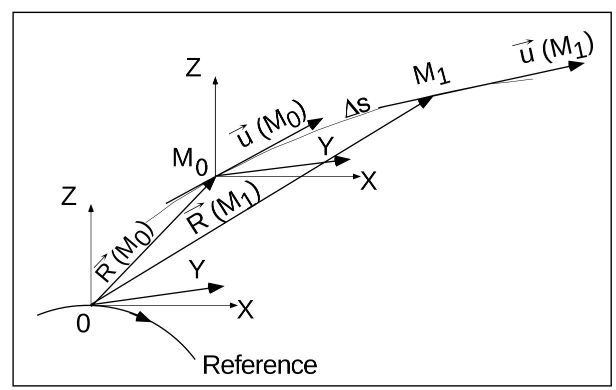
Introduce reduced notations, $\vec{u}=\frac{\vec{v}}{v},\quad \vec{B}=\frac{\vec{b}}{B\rho}$, **then :**

$$\boxed{\vec{u}\,' = \vec{u}\times\vec{B}}$$

- **Solved using truncated Taylor expansions of $\vec{R}$ and $\vec{u}=\vec{v}/v$:**

$$\vec{R}(M_1)\approx\vec{R}(M_0)+\vec{u}(M_0)\,\Delta s+\vec{u}\,'(M_0)\,\frac{\Delta s^2}{2!}+\ldots+\vec{u}\,'''''(M_0)\,\frac{\Delta s^6}{6!}$$

$$\vec{u}(M_1)\approx\vec{u}(M_0)+\vec{u}(M_0)\,\Delta s+\vec{u}\,''(M_0)\,\frac{\Delta s^2}{2!}+\ldots+\vec{u}\,'''''(M_0)\,\frac{\Delta s^5}{5!}$$



• **Over 40+ years... oodles of magnetic elements have been installed**

What you want to simulate :	*Keyword :*
Semi-analytical models :	
Decapole	DECAPOLE, MULTIPOL
Dipole(s), spectrometer	BEND, DIPOLE[S][-M], MULTIPOL, QUADISEX
Dodecapole	DODECAPO, MULTIPOL
FFAG magnets	DIPOLE[S], FFAG, FFAG-SPI
Multipole	MULTIPOL, QUADISEX, SEXQUAD
Octupole	OCTUPOLE, MULTIPOL, QUADISEX, SEXQUAD
Quadrupole	QUADRUPO, MULTIPOL, SEXQUAD
Sextupole	SEXTUPOL, MULTIPOL, QUADISEX, SEXQUAD
Solenoid	SOLENOID
Helical dipole	HELIX
Field maps :	
1-D, cylindrical symmetry	BREVOL
2-D, mid-plane symmetry	CARTEMES, POISSON, TOSCA
2-D, no symmetry	MAP2D
2-D, polar mesh	POLARMES
3-D	TOSCA
4-D : time !	

AN EXAMPLE OF A "KEYWORD" : 'MULTIPOL'

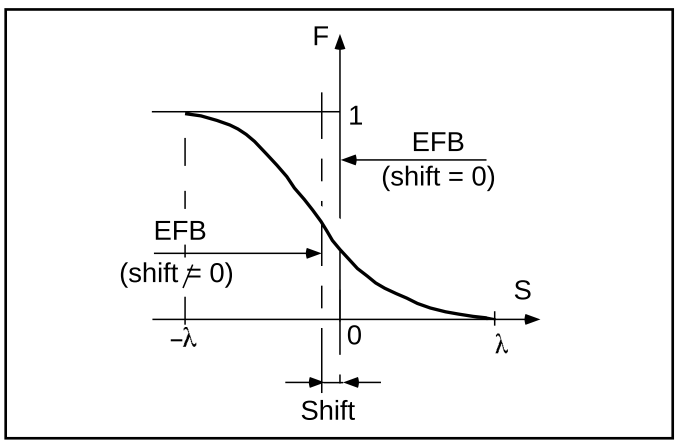
- **Field and derivatives as needed in the Taylor series**

$$\frac{\partial^{i+j+k}\vec{B}_n(X,Y,Z)}{\partial X^i \partial Y^i \partial Z^k} \qquad i+j+k = 0 \textbf{ to } 4 \tag{1}$$

are obtained by differentiation of the scalar potential

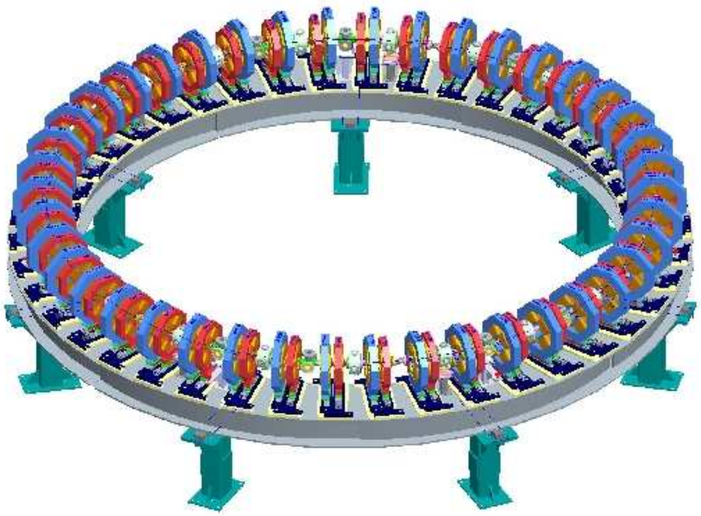
$$V_n(X,Y,Z) = (n!)^2 \left(\sum_{q=0}^{\infty} (-1)^q \frac{G^{(2q)}(X)(Y^2+Z^2)^q}{4^q q!(n+q)!} \right) \left(\sum_{m=0}^{n} \frac{sin\left(m\frac{\pi}{2}\right) Y^{n-m} Z^m}{m!(n-m)!} \right) \tag{2}$$

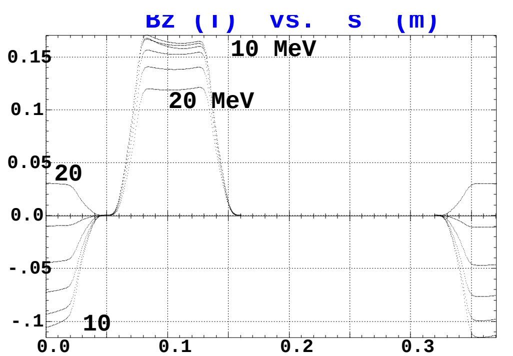
- $G(s)$ **is a longitudinal form factor which simulates the "field fall-off"**

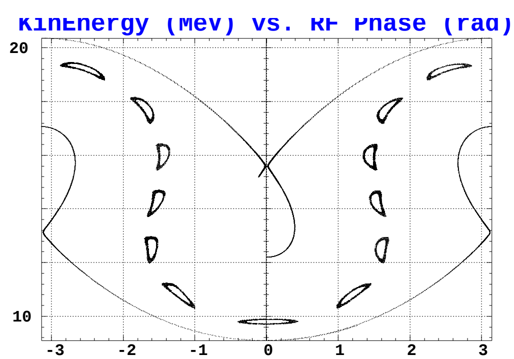


EXAMPLE (2005+) – Virtual EMMA FFAG / ON-LINE MODEL

(several companion posters and papers at CYC'13)



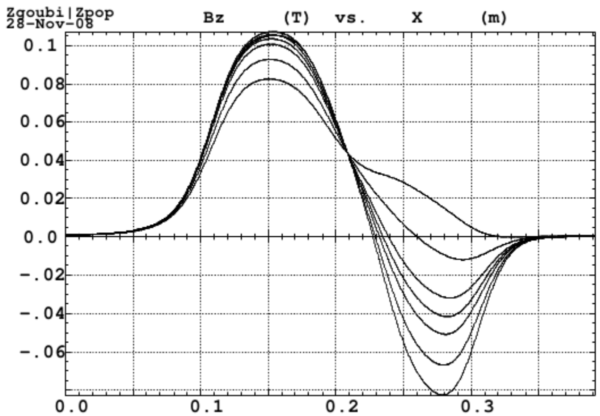




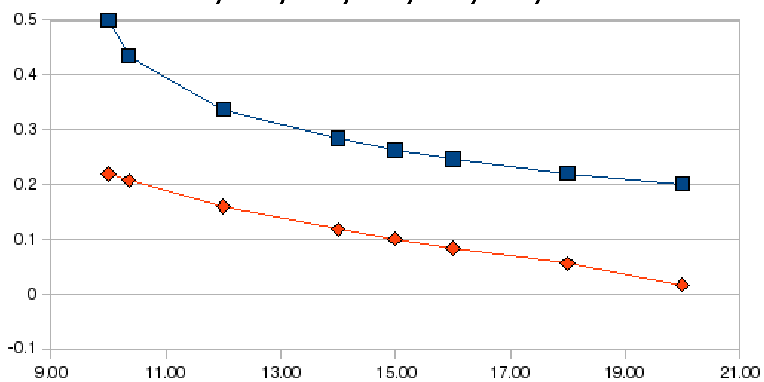
Zgoubi input data file - excerpt :

```
'MARKER' RingInj BegRing                  start of ring. Injection point
'MULTIPOL' QD                                        start of first cell
0
7.5698 5.3 0. -2.49324 0 0 0 0 0 0 0 0
1.00 1.00 1.00 1.00 1.00 1. 1. 1. 1.
4 .1455 2.2670 -.6395 1.1558 0. 0. 0.
1.00 1.00 1.00 1.00 1.00 1. 1. 1. 1.
4 .1455 2.2670 -.6395 1.1558 0. 0. 0.
0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
0.1
2 0. 3.404834122312866 0.
'MARKER' BPM2 off                                           BPM location
'DRIFT' sd
5.00
'MULTIPOL' QF
0
5.8782 3.7 0. 2.47708 0 0 0 0 0 0 0 0
1.00 1.00 1.00 1.00 1.00 1. 1. 1. 1.
4 .1455 2.2670 -.6395 1.1558 0. 0. 0.
1.00 1.00 1.00 1.00 1.00 1. 1. 1. 1.
4 .1455 2.2670 -.6395 1.1558 0. 0. 0.
0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
0.1
2 0. 0.7513707181808552 0.
'DRIFT' ld
8.
'CAVITE'                                         programmable RF cavity
7
0.736669 1.3552e9                          Orbit length, RF frequency
70e3 0.                                       Voltage, relative phase
'MARKER' BPM1 off                                           BPM location
'CHANGREF'                             cell orientation - wrt. next one
0. 0. -8.571428571429                                  end of first cell
————————-
next 41 cells
————————-
'REBELOTE'                                          multiturn tracking
150 0.2 99
'END'
```

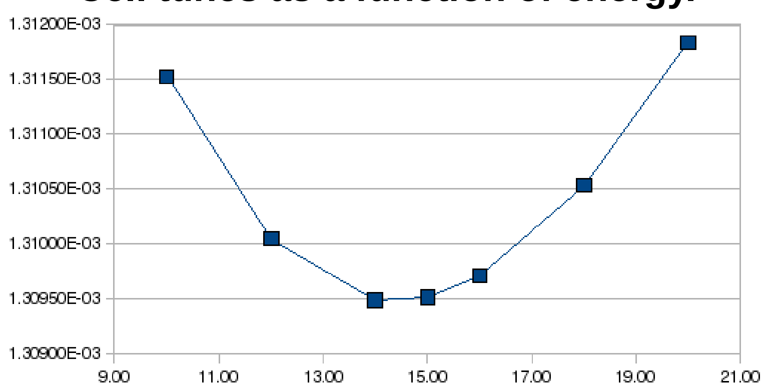
EMMA cell, using field map ’02611DF.table’ / compare with ’DIPOLES’



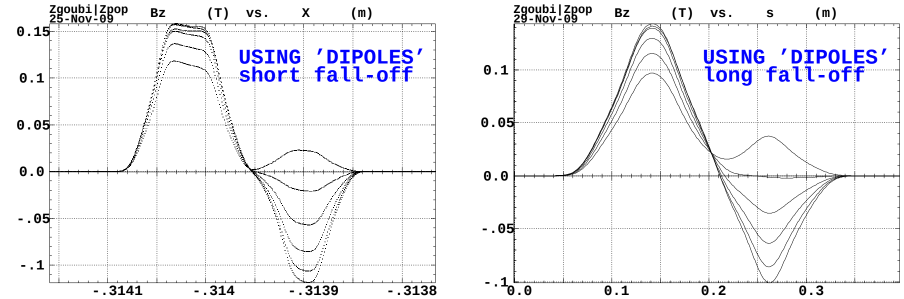
Field on orbits, 10, 12, 14, 15, 16, 18 and 20 MeV.



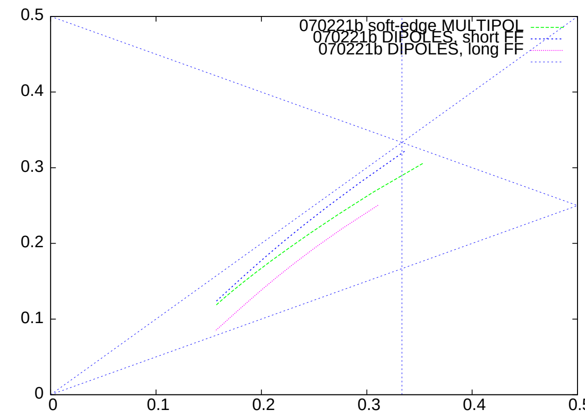
Cell tunes as a function of energy.



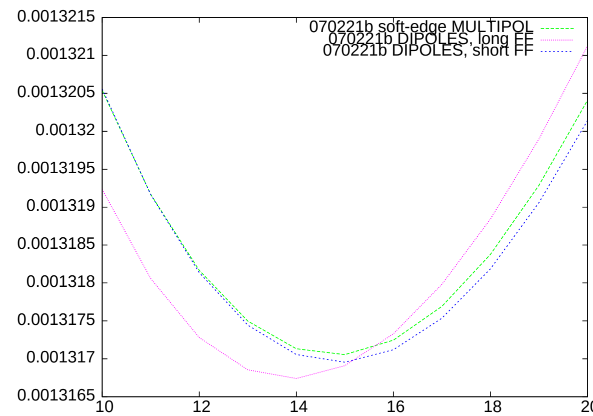
Time of flight as a function of energy.



Field on orbits, 10, 12, 14, 15, 16, 18 and 20 MeV.



Cell tunes as a function of energy.



Time of flight as a function of energy.

NOTE : 'TOSCA' WILL SUPERIMPOSE FIELD MAPS FOR YOU :

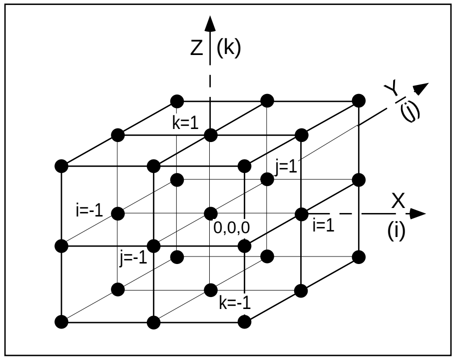
Zgoubi input data file - excerpt :

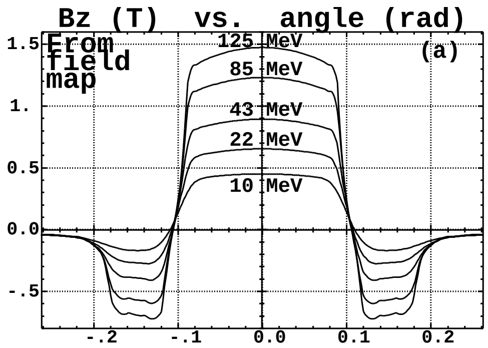
```
EMMA CELL, USING FIELD MAPS
'OBJET'
+5.171103865921708e+01
2
1 1
-4.83 1.38E+02 0. 0. 0. 0.7 '0'
1
'PARTICUL'
0.51099892 1.60217653e-19 0.0 0.0 0.0
'PICKUPS'
1
#E
'FAISTORE'
b_zgoubi.fai #E
1
'TOSCA'
0 0
-10. .1 .1 .1
QPOLES HEADER_8
961 161 1 15.2 1. 1.                 IZ=1 & mod.mod2=15.2 ⟹ two 2D maps / up to 5
b_both-20130204a-f-off.table
b_both-20130204a-d-off.table
0 0 0 0
2
.1
2 0 0 0
'CHANGREF'
0. 0. -8.57142857152
'FAISCEAU'
'MARKER' #E
'MATRIX'
1 11
'FAISCEAU'
'END'
```

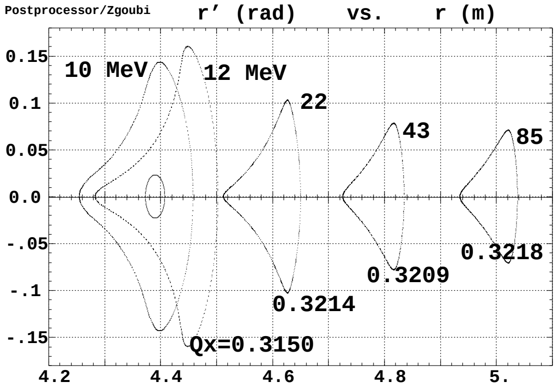
EXAMPLE (∼2005) - 'FFAG' and 'TOSCA' keywords

A simulation of a 10-cell 150 MeV FFAG ring based on a scaling FFAG dipole triplet cell.

- **Using 'TOSCA' keyword and an OPERA field map of the dipole triplet :**

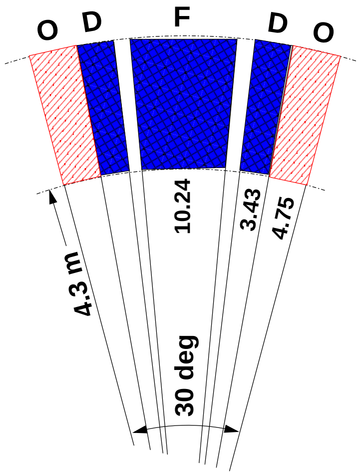


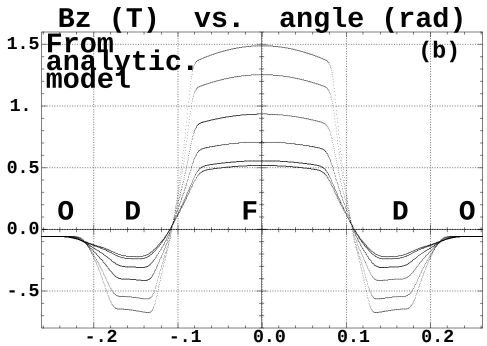


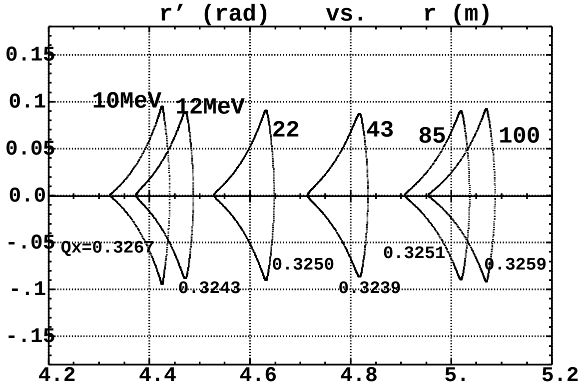


- **Using 'FFAG' keyword ('DIPOLES' would do as well) : superposition of dipole fields** [NIM A 547, Lemuet, Méot]

$$B_z(r,\theta) = \sum_{i=1,N} B_{z0,i}\, \mathcal{F}_i(r,\theta)\, \mathcal{R}_i(r)$$







Exemples available in sourceforge → code → exemples/KEK150MeVFFAG/analyticalModel, ./OPERAMap.

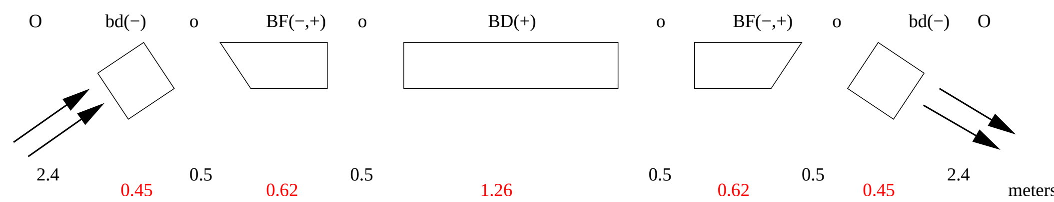
Zgoubi input data file, analytical model :

```
FFAG triplet. 150MeV machine
'OBJET'
1839.090113 150MeV
5
0.001 0.001 0.001 0.001 0.001 0.0001
486.802 0. 0.0 0. 0. 0.562925 50MeV
'FFAG' #START
0
3 30. 540. 1
6.465 0. -14.308348 7.25 0. 0.                  DIPOLE #1 : ACNT, dum, B0, K,dummies
4. 000                                          EFB 1 : lambda, iop=data option f
4 .1455 2.2670 -.6395 1.1558 0. 0. 0.
1.715 0. 1.E6 -1.E6 1.E6 1.E6
4. 000                                          EFB 2
4 .1455 2.2670 -.6395 1.1558 0. 0. 0.
-1.715 0. 1.E6 -1.E6 1.E6 1.E6
0. -1                                           EFB 3 : inhibited by iop=0
0 0. 0. 0. 0. 0. 0. 0.
0. 0. 0. 0. 0. 0.
15. 0. 17.16 7.58 0. 0.                         DIPOLE #2 : ACNT, dum, B0, K,dummies
4. 000                                          EFB 1
4 .1455 2.2670 -.6395 1.1558 0. 0. 0.
5.12 0. 1.E6 -1.E6 1.E6 1.E6
4. 000                                          EFB 2
4 .1455 2.2670 -.6395 1.1558 0. 0. 0.
-5.12 0. 1.E6 -1.E6 1.E6 1.E6
0. -1                                           EFB 3
0 0. 0. 0. 0. 0. 0. 0.
0. 0. 0. 0. 0. 0.
23.535 0. -14.308348 7.25 0. 0.                 DIPOLE #3 : ACNT, dum, B0, K,dummies
4. 000                                          EFB 1
4 .1455 2.2670 -.6395 1.1558 0. 0. 0.
1.715 0. 1.E6 -1.E6 1.E6 1.E6
4. 000                                          EFB 2
4 .1455 2.2670 -.6395 1.1558 0. 0. 0.
-1.715 0. 1.E6 -1.E6 1.E6 1.E6
0. -1                                           EFB 3
0 0. 0. 0. 0. 0. 0. 0.
0. 0. 0. 0. 0. 0.
2 4.                                            IRD(=2, 25 or 4), resol(=¿step/*)
.5                                              integration step size (cm)
2 0. 0. 0. 0.
'MATRIX'
1 11
'END'
```

Zgoubi input data file, OPERA map :

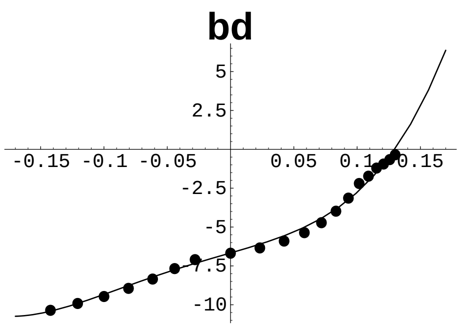
```
150MeV FFAG
'OBJET'
1839.090113 150MeV
5
.001 .001 .001 .001 .001 .0001
444.15 0. 0.0 0. 0. 0.273042677097 'o'    12MeV Brho=502.1500877
'TOSCA' #START
0 20
-1.e-3 1. 1. 1.
HEADER_8 FFAG 150MeV
301 121 41 20
k75v113my021f45500d2700.table             OPERA field map
0 0 0 0
2
.0125
2
0. 0. 0. 0.
'MATRIX'
1 11
'END'
```

ISOCHRONOUS MUON FFAG
(G. REES, ∼2004)



• **bd and BD multipoles :**

Gradient profiles K (m^{-2}) vs. x (m)



The magnets' gradients are constitutive of the design data, they are approximated using 4th degree polynomials.

$$B_{bd}(x) = -6.66771 + 23.5565r\, x + 11.9699\, x^2 + 926.188\, x^3 + 4952.98\, x^4$$
$$B_{BD}(x) = -9.723 - 51.5803\, x - 697.091\, x^2 - 33956.1\, x^3 - 241808\, x^4$$

The gradients are integrated to get the multipole coefficients of the field.

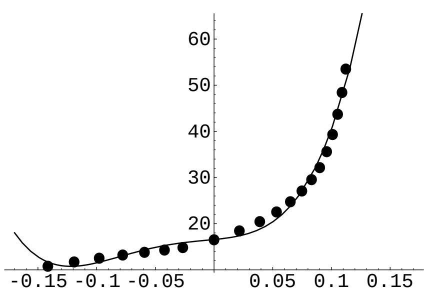
$$B_{bd}(x) = \mathbf{b_{bd0}} - 6.66771\, x + 11.7783\, x^2 + 3.98996\, x^3 + 231.547\, x^4 + 990.6\, x^5 \qquad (3)$$
$$B_{BD}(x) = \mathbf{b_{BD0}} - 9.723\, x - 25.7902\, x^2 - 232.364\, x^3 - 8489.03\, x^4 - 48361.5\, x^5 \qquad (4)$$

After finding the dipole cofficients $\mathbf{b_{bd0}}$, $\mathbf{b_{BD0}}$ with the matching procedure (next slide), the field and its derivatives are derived from the classical multipole modelling of the form.

$$\vec{B} = \mathbf{grad}V_n \ \ \textbf{with}\ \ V_n(s,x,z) = (n!)^2 \left(\sum_{q=0}^{\infty} \frac{(-)^q\, G^{(2q)}(s)(x^2+z^2)^q}{4^q q!(n+q)!} \right) \left(\sum_{m=0}^{n} \frac{sin\left(m\frac{\pi}{2}\right) x^{n-m} z^m}{m!(n-m)!} \right) \qquad (5)$$

with $G^{(2q)}(center)$ representing the (derivatives of) the fringe field form factor.

• **BF sector magnet :**



The same gradient matching procedure is applied to obtain

$$B_{BF}(r) = \mathbf{b_{BF0}} + 16.5655\, r + 12.612\, r^2 + 86.4359\, r^3 + 2987.43\, r^4 + 13647.1\, r^5$$

Transform from BF cylindrical frame into Zgoubi Cartesian frame, using

$$\partial B_z/\partial X = (1/r)\partial B_z/\partial\theta, \ \ \partial B_z/\partial Y = \partial B_z/\partial r, \ \ \partial^2 B_z/\partial X^2 = (1/r^2)\partial^2 B_z/\partial\theta^2 + (1/r)\partial B_z/\partial r, \ \ etc.$$

Z-derivatives and extrapolation off mid-plane yield the 3-D $\vec{B}$ model

$$\vec{B}(X,Y,Z) \quad , \quad \partial^{i+j+k}\vec{B}/\partial X^i \partial Y^j \partial Z^k$$

Zgoubi input data

```
'MULTIPOL'      bd
  00
  45 100.00 -3.45374050E+01   -66.6771 117.783 39.8996 2315.47 9905.97 0. 0.0 0.0 0.0
 0. 0. 9. 4.  1.00 1.00 1.00 1.00 1.00 1. 1. 1. 1.
4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 0. 0. 9. 4.  1.00 1.00 1.00 1.00 1.00 1. 1. 1. 1.
4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
.5 step  bd
1 0. 0. 0.
'DRIFT'
50.
 'DIPOLES'    BF
  00
 1  1.463414634   24.274311920375e2           nbmag  AT/deg, RM/cm
 0.731707317  0. -2.25637704 5 -1782.12892 -32935.7018 -5479274.31  -4.59698831E+09 -5.09757397E+11
 0. 0.                                       EFB 1
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 0.731707317    0.  1.E6  -1.E6  1.E6  1.E6
 0. 0.                                       EFB 2
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
-0.731707317    0.   1.E6  -1.E6  1.E6  1.E6
 0. 0.                                        EFB 3
 0  0.       0.       0.       0.       0. 0.  0.
 0.  0.   0.    0.    0. 0. 0.
 0    2  64.
 .5 step  BF                                         step
 2   2.42294098E+03     0. 2.43432058E+03  0.   24.228e2  24.3458e2
'DRIFT'
50.
'MULTIPOL'      BD
  00
  63. 100.00  4.21503506E+01  -97.23 -257.902 -2323.64 -84890.3 -483615 0. 0. 0. 0.
 0. 0. 9. 4.  1.00 1.00 1.00 1.00 1.00 1. 1. 1. 1.
4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 0. 0. 9. 4.  1.00 1.00 1.00 1.00 1.00 1. 1. 1. 1.
4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
.5 step  BD
1 0. 0. 0.
```

Correlation of beam losses and tunes

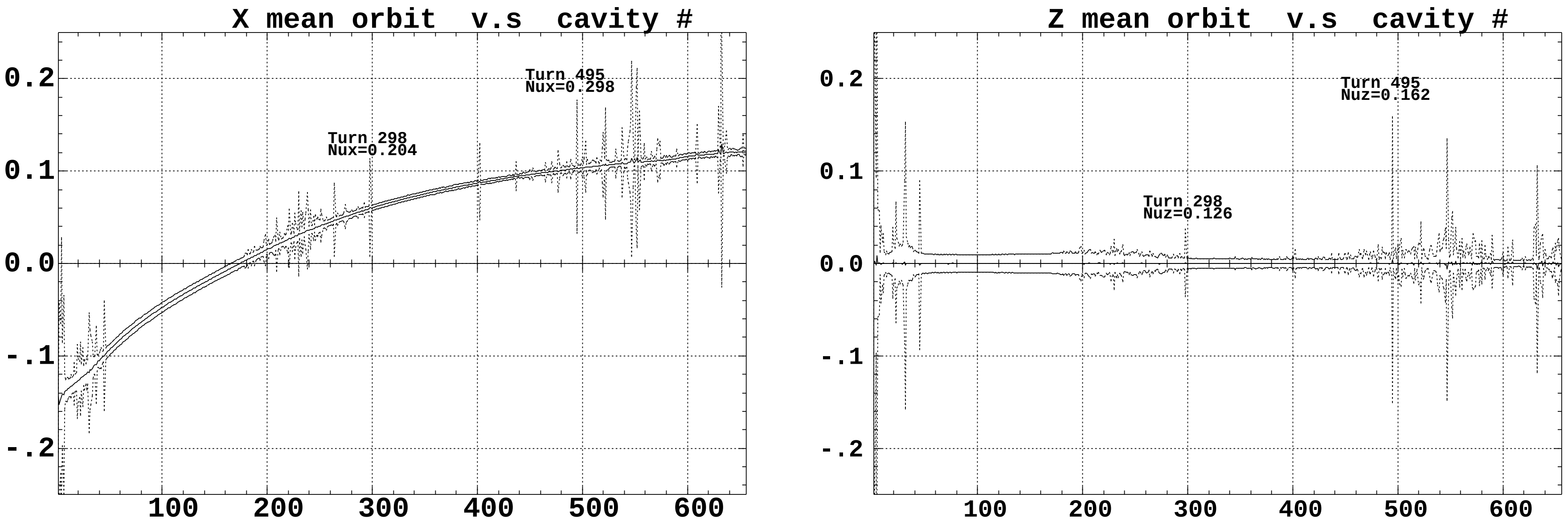
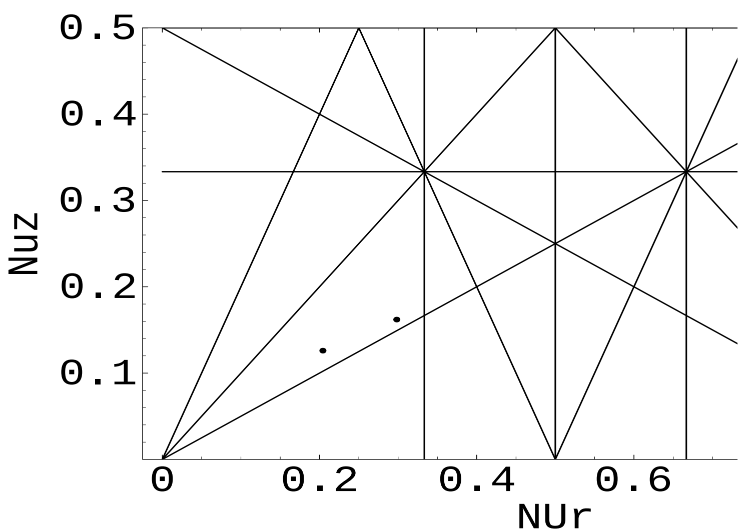
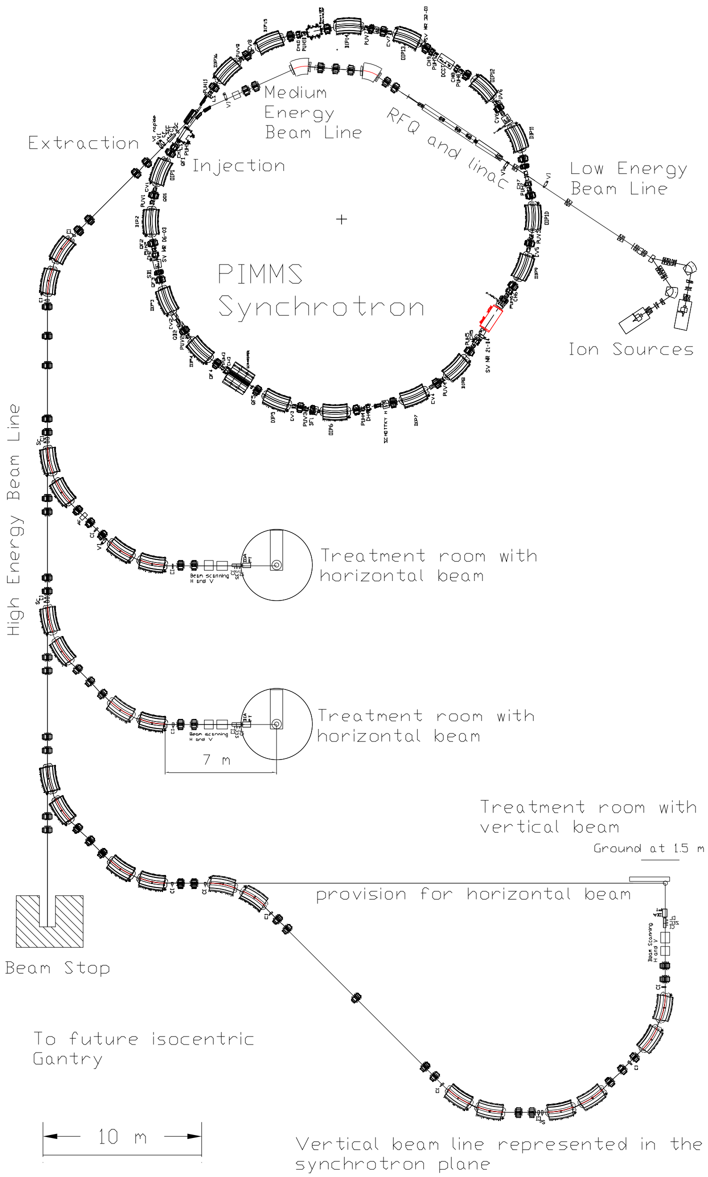


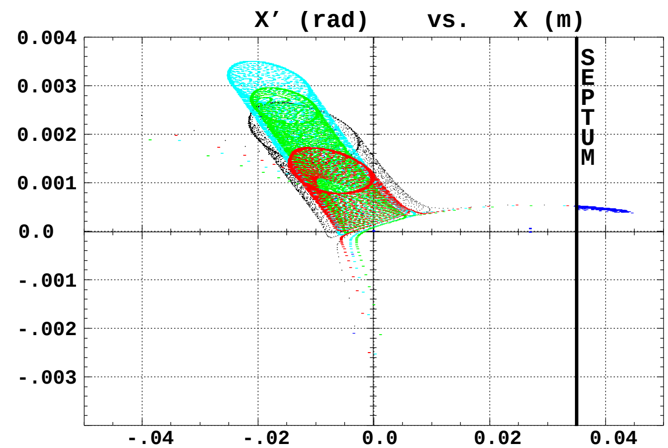
Figure 1:

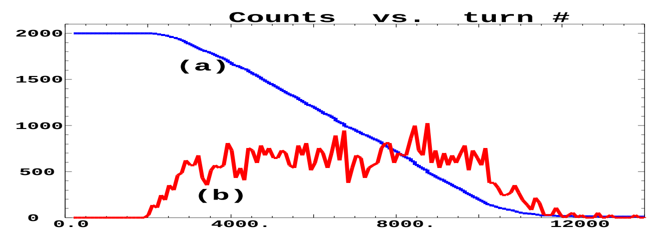


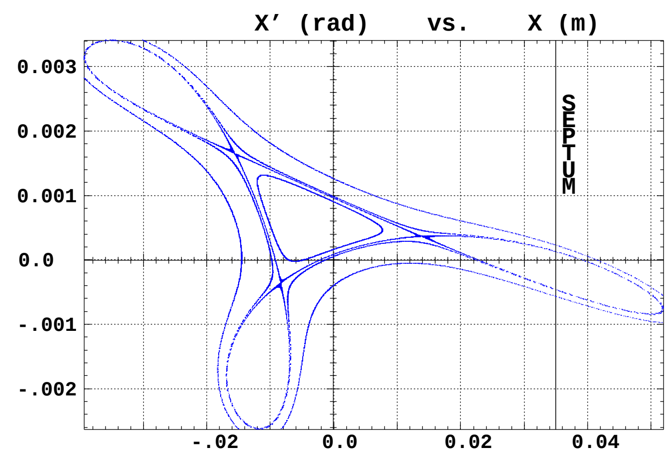
EXAMPLE ($\sim$2000) – SLOW EXTRACTION

- Simulation of resonant slow extraction from a carbon synchrotron
- **Main difficulties : (i) motion near separatrix, (ii) slow process, $\sim$0.1 second(s) $\Rightarrow$ $\boxed{>> 10^5 \text{ turns tracking}}$.**





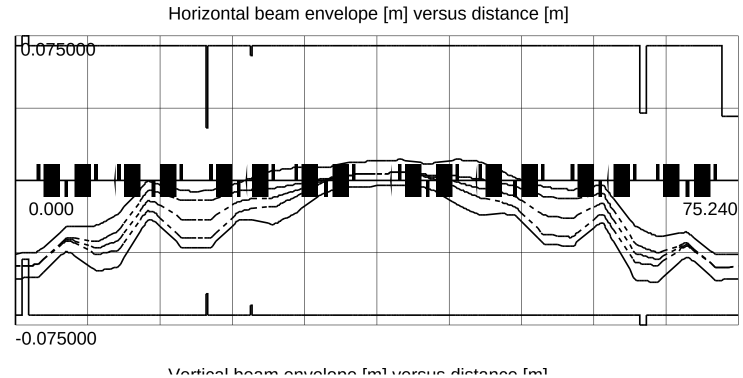




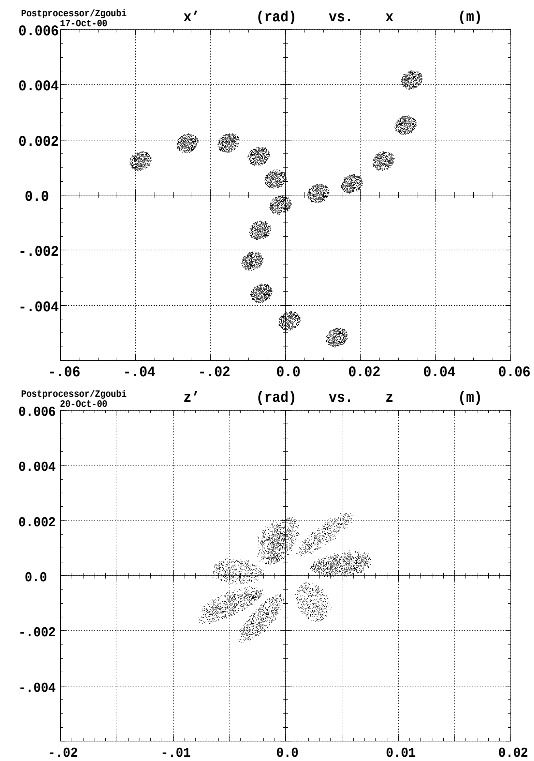
Zgoubi input data file, using 'SCALING'

```
****** Extraction, C6+
'MCOBJET'      Monte Carlo object, C6+ 120MeV/u
1000.
3
40
2 2 1 1 1 1
0. 0. 0. 0. 0. 0.997
0. 8.562 7.143E-6 1
0. 2.848 7.143E-6 1
0. 1. 4.E-6 1
123456 234567 345678

'SCALING'
1 2
MULTIPOL XR_
3
0 1 1
1 2000 999999
BETATRON
4
0. 0. 1 1
1 2000 2001 999999

'FAISTORE' lmnt# 106 179 206
b_xtract4Dp.fai ESE_ICOL
1
'BETATRON'      Start of ring
2.5e-6
'DRIFT' DRIF SS_MR_01_03
75.0000
'COLLIMA' ESi_COL
1
1 5.8 3.7 1.7 0. 1.7-5.8=-4.1, 1.7+5.8=7.5
'DRIFT' DRIF ES_INJECTION
60.0000
————————
rest of the ring
————————
'DRIFT' DRIF SS_MR_01_02
109.4000
'FAISCEAU'      End of ring
'REBELOTE'
99999 0.1 99
'END'
```

• 16-turn injection, 'SCALING' commands the orbit bump



Horizontal injection bump.



Carbon injection, 16 turns injected, observed at injection septum.

3 THE ELECTRIFICATION OF ZGOUBI

... intervened in the early 1990s, motivated, as usual, by on-going R/D tasks.

- **When both $\vec{e}$ and $\vec{b}$ are non-zero, the complete equation is solved,**

$$\boxed{(B\rho)'\vec{u} + B\rho\,\vec{u}' = \vec{e}\,/\,v \;+\; \vec{u}\times\vec{b}}$$

One can then push the rigidity, with the same method of (truncated) Taylor series

$$(B\rho)(M_1) \approx (B\rho)(M_0) + (B\rho)'(M_0)\Delta s + ... + (B\rho)''''(M_0)\frac{\Delta s^4}{4!} \tag{6}$$

and the time of flight,

$$T(M_1) \approx T(M_0) + \frac{dT}{ds}(M_0)\,\Delta s + \frac{d^2T}{ds^2}(M_0)\,\frac{\Delta s^2}{2} + \frac{d^3T}{ds^3}(M_0)\,\frac{\Delta s^3}{3!} + \frac{d^4T}{ds^4}(M_0)\,\frac{\Delta s^4}{4!} \tag{7}$$

- **A list of the electrostatic elements :**

What you want to simulate :	*Keyword :*
Semi-analytical models :	
2-tube (bipotential) lens	EL2TUB
3-tube (unipotential) lens	UNIPOT
Decapole	ELMULT
Dipole	ELMULT
Dodecapole	ELMULT
Multipole	ELMULT
N-electrode mirror/lens, straight slits	ELMIR
N-electrode mirror/lens, circular slits	ELMIRC
Octupole	ELMULT
Quadrupole	ELMULT
R.F. (kick) cavity	CAVITE
Sextupole	ELMULT
Skewed multipoles	ELMULT
Field maps :	
1D, cylindrical symmetry	ELREVOL
2-D, no symmetry	MAP2D_E

• A list of the magneto-electrostatic elements :

What you want to simulate :	*Keyword :*
Semi-analytical models :	
Decapole	EBMULT
Dipole	EBMULT
Dodecapole	EBMULT
Multipole	EBMULT
Octupole	EBMULT
Quadrupole	EBMULT
Sextupole	EBMULT
Skew multipoles	EBMULT
Wien filter	SEPARA, WIENFILT

EXAMPLE (TRIUMF, 1990) - TWO-STAGE 800-MeV/c KAON BEAMLINE, USING TWO *WIEN-FILTERS*

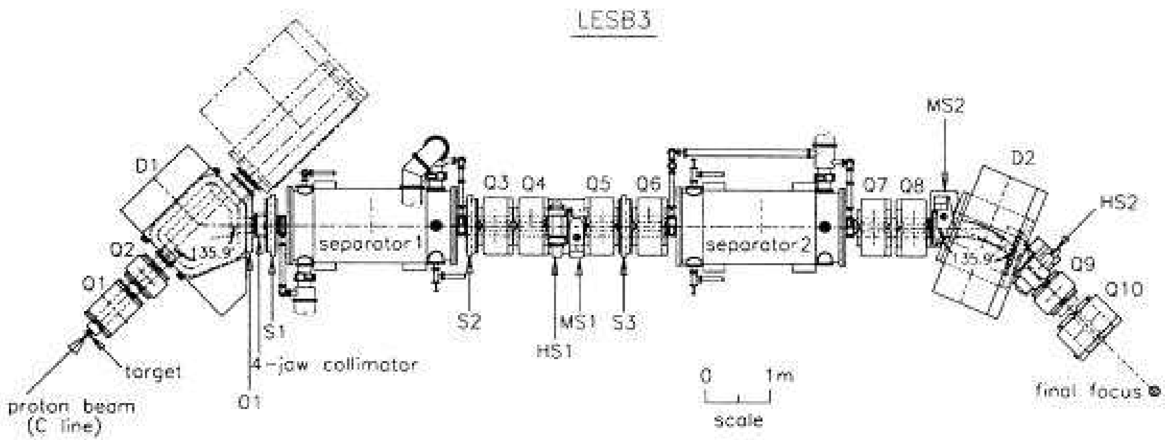


Fig. 1. Lay out of LESB3 beamline.

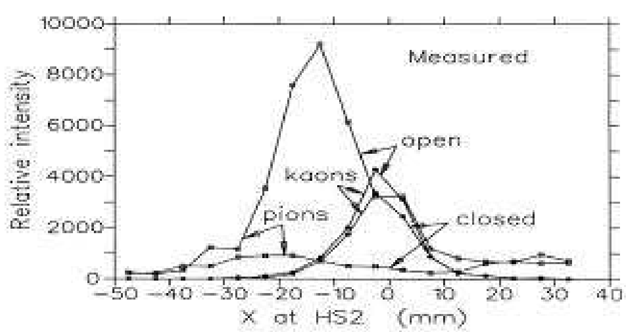


Fig. 16. Measured kaon and pion distributions at HS2 with the four-jaw collimator open and upper-right jaw closed. Compare with Fig. 10(a).

4 SPIN TRACKING

... was installed in 1990 for a partial siberian snake project at the 3 GeV ring SATURNE, Saclay.

- **Equation of spin precession :**

$$\frac{d\vec{S}}{dt} = \frac{q}{m}\vec{S}\times\vec{\Omega}, \quad \textbf{with} \quad \vec{\Omega} = (1+\gamma G)\vec{b} + G(1-\gamma)\vec{b}_{//}$$

- **Normalize as earlier**

$$\boxed{\vec{S}\,' = \vec{S}\times\vec{\omega}}$$

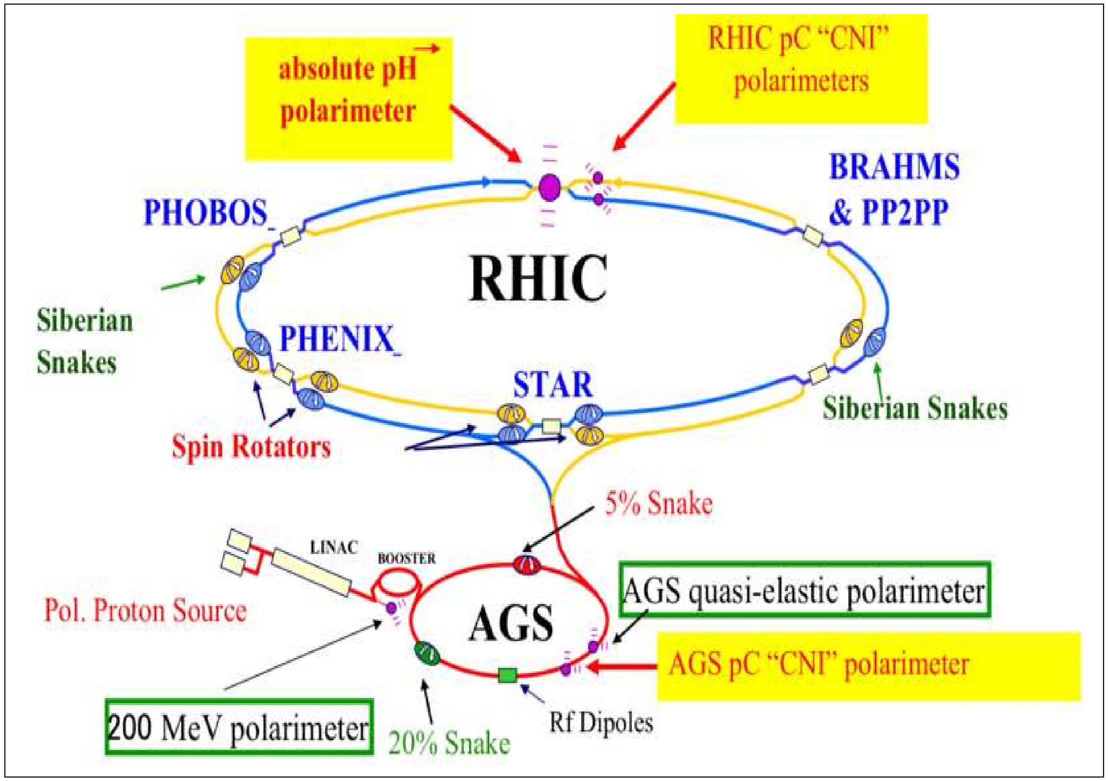
same form as $\vec{u}' = \vec{u}\times\vec{B}$ **!**

It is solved using the outcomes of the particle ray-tracing.

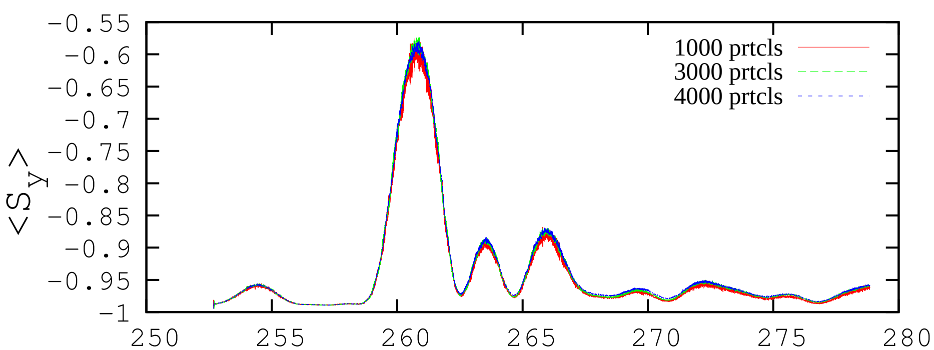
- **use again truncated Taylor expansion to push** $\vec{S}$

$$\vec{S}(M_1) \approx \vec{S}(M_0) + \frac{d\vec{S}}{ds}(M_0)\,\Delta s + \frac{d^2\vec{S}}{ds^2}(M_0)\frac{\Delta s^2}{2} + \frac{d^3\vec{S}}{ds^3}(M_0)\frac{\Delta s^3}{3!} + \frac{d^4\vec{S}}{ds^4}(M_0)\frac{\Delta s^4}{4!}$$

EXAMPLE (2009...) - RHIC STUDIES - GROUND FOR eRHIC

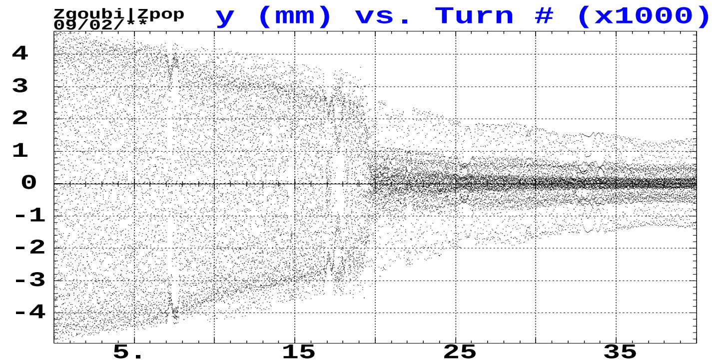


Polarization studies in RHIC - 10^5 turn runs :



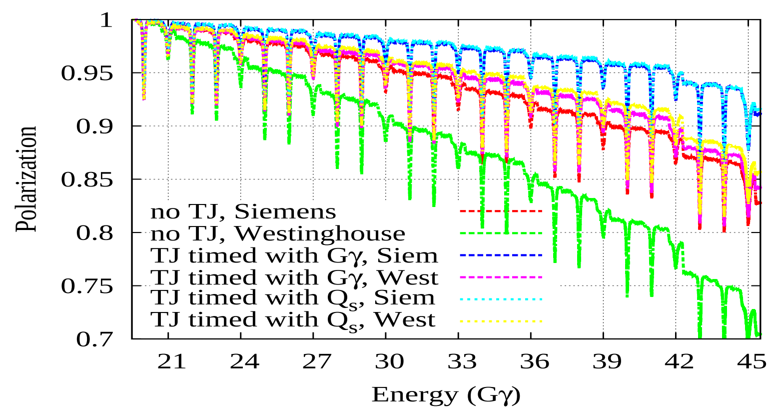
Average polarization as a function of energy at traversal of the snake resonance $G\gamma = 231 + Q_y$.

BD studies in the AGS with snakes :



Horizontal excursion from injection to transition energy. 5 particles.
~40000 turns, 20 min. CPU.

Optimization of polarization transmission :



3000 particles tracking, 40000 turns.
Exploring machine setting conditions.

5 SYNCHROTRON RADIATION - ENERGY LOSS

Was installed in ∼2000 for emittance increase studies along the linear collider BDS.

- **The energy loss is calculated after each integration step Δs, in a classical manner, accounting for two random processes :**

- probability of emission of a photon

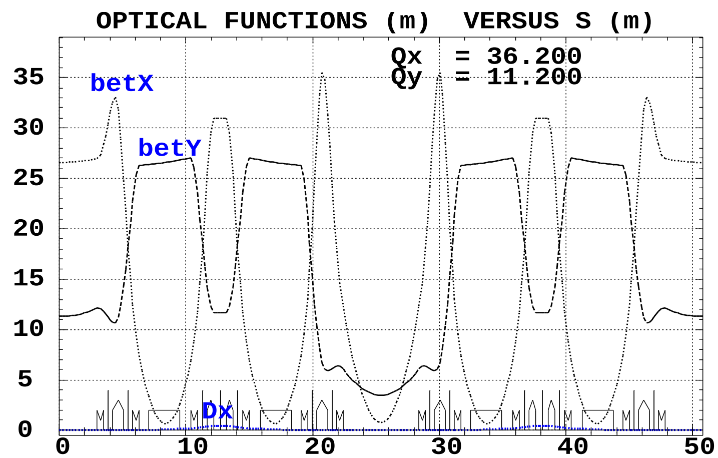
- probability of the photon energy

EXAMPLE (2009) – SYNCHROTRON RADIATION DAMPING IN RINGS

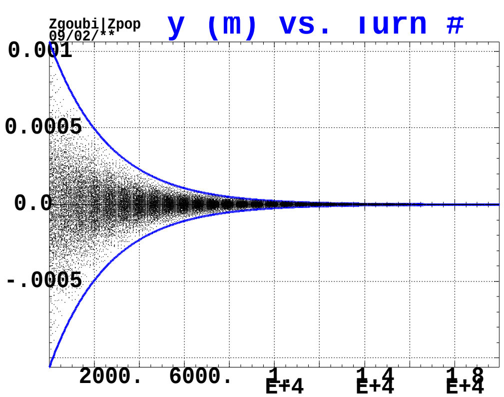
Consider ESRF Chasman-Green super-cell.

Interest : all-analytical understanding.

16 cells ring, 812.6 m, 64 bends.



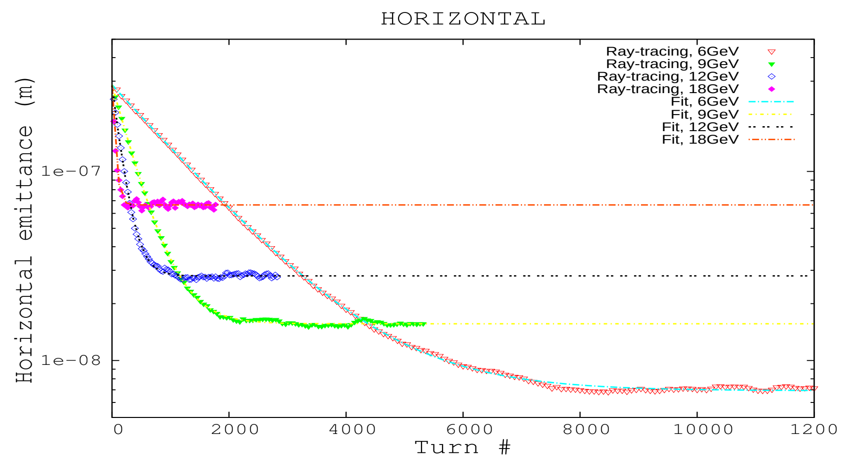
Principle :



Damping of vertical motion over 20000 turns (left), single particle is tracked. Its vertical invariant (right) decreases towards zero.

Emittance damping :

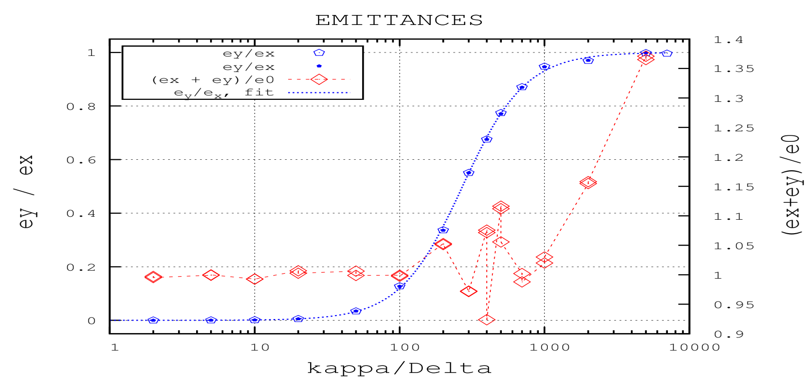
$$\epsilon(t) = \epsilon_0\, e^{-t/\tau} + \epsilon_{equil.}\left(1 - e^{-t/\tau}\right)$$



$\tau_x \approx 1300$ **turns.**

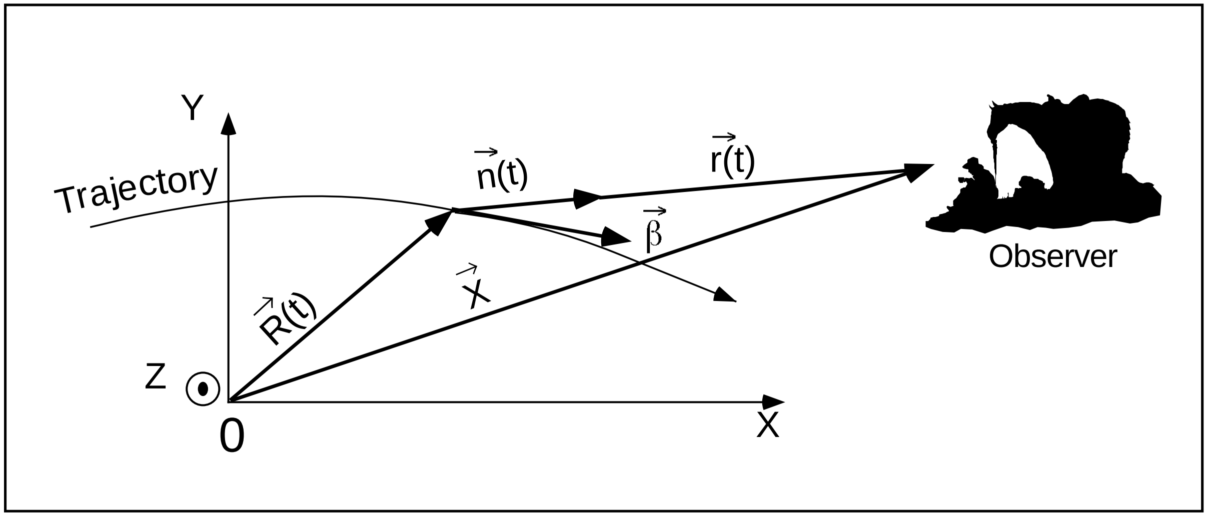
Coupling :

$$\frac{\epsilon_y}{\epsilon_x} = \frac{\kappa^2}{\kappa^2 + \Delta^2}, \qquad \epsilon_x + \epsilon_y = \epsilon_0.$$



6 SYNCHROTRON RADIATION - SPECTRAL-ANGULAR DENSITY

- **Was installed in 1994 for the study of deleterous interference effects at the LEP beam diagnostics mini-wiggler.**



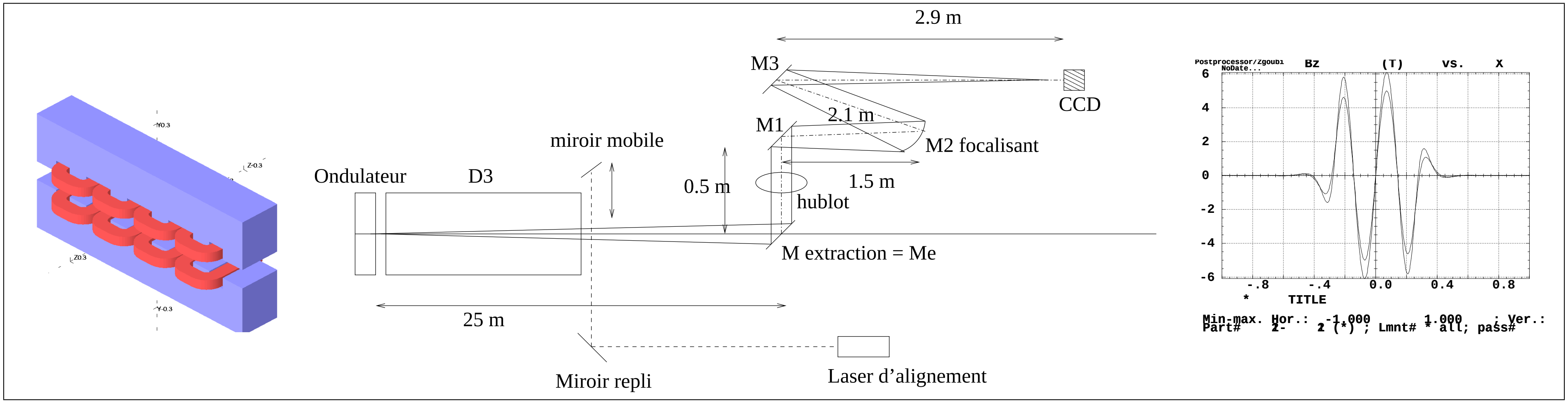
- **The ray-tracing provides the ingredients to compute**

$$\vec{\mathcal{E}}(\vec{n},\tau) = \frac{q}{4\pi\varepsilon_0 c}\,\frac{\vec{n}(t)\times\left[\left(\vec{n}(t)-\vec{\beta}(t)\right)\times d\vec{\beta}/dt\right]}{r(t)\left(1-\vec{n}(t)\cdot\vec{\beta}(t)\right)^3}, \qquad \mathcal{B} = \vec{n}\times\vec{\mathcal{E}}/c$$

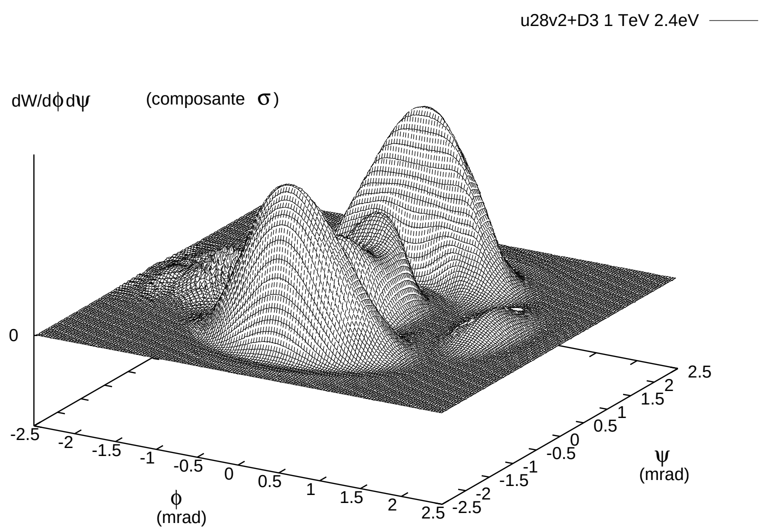
- **The electric field of the radiation is then Fourier transformed, so yielding the spectral angular energy density :**

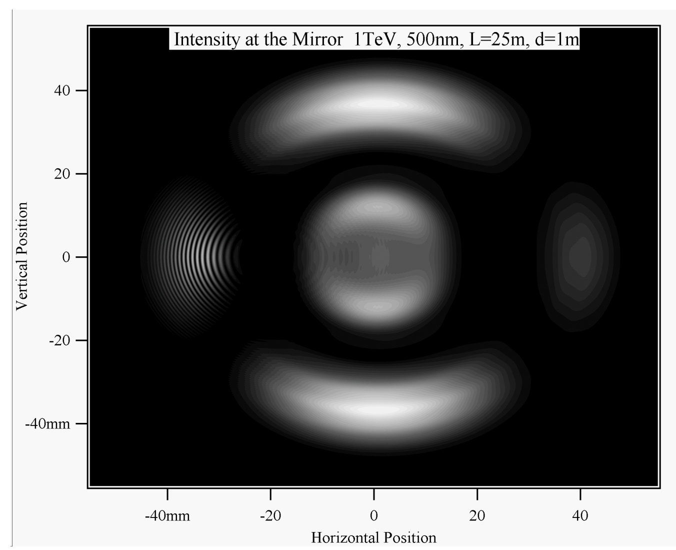
$$\partial^3 W/\partial\phi\,\partial\psi\,\partial\omega = 2r^2\left|FT_\omega\left(\vec{\mathcal{E}}(\tau)\right)\right|^2/\mu_0 c$$

EXAMPLE (2000) – DESIGN OF DIAGNOSTICS INSTALLATIONS AT LHC



• **LHC undulator is against a long dipole. The optical system is drawn from LEP's.**





• **Intensity emitted (horizontal component) by 1 TeV protons, λ = 500 nm, with a distance d = 1 m between the two sources, simulated with Zgoubi (left) and with SRW (right).**

7 SPIN DIFFUSION

... a spin-off ! **Comes for free**

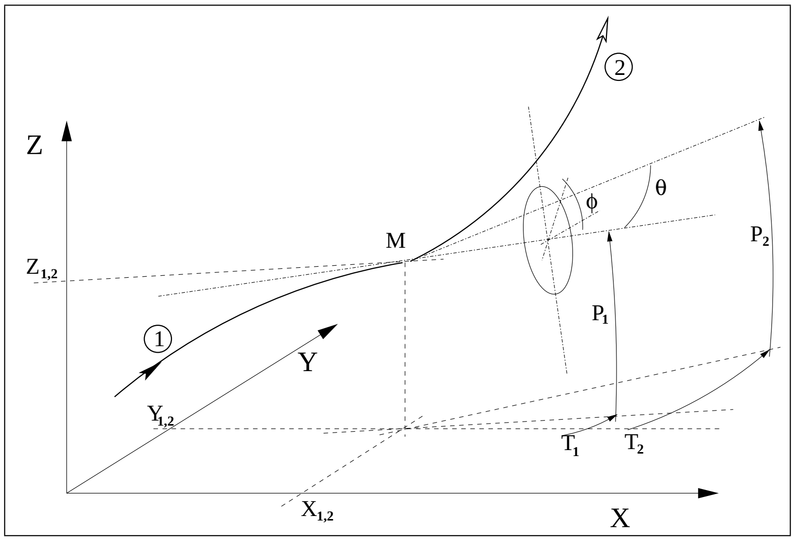
$$\left.\begin{array}{r}\textbf{SPIN DYNAMICS}\\ \textbf{+}\\ \textbf{STOCHASTIC ENERGY LOSS BY SR}\end{array}\right\}\Longrightarrow \textbf{SPIN DIFFUSION}$$

We are working on that, at the moment,

in relation with the eRHIC project R/D studies at BNL.

8 IN-FLIGHT DECAY

... was installed for eta meson spectrometry at SATURNE, Saclay, late 1980s.



Parent particle, kinematics ingredients needed :

- **lifetime** $\tau_\pi = \gamma_\pi \tau^*_\pi$,
- **decay law** $N(s) = N_0 e^{-\eta s/p_\pi}$ ($\eta = m_\pi \;/\; c\tau^*_\pi$)
- **momentum,** $\vec{p_\pi}$

Daughter particle, kinematics ingredients then derived :

- ***com* energy** $E^*_\mu = (m^2_\pi + m^2_\mu)/2m_\pi$
- **momentum** $\vec{p^*_\mu} = (m^2_\pi - m^2_\mu)/2m_\pi \; \vec{u}$,
- **lab. energy** $E_\mu = \gamma_\pi (E^*_\mu + \beta_\pi p^*_\mu \cos\theta^*_\mu)$

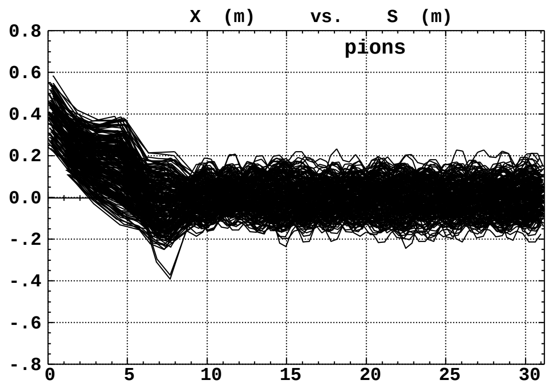
Monte Carlo procedure sorts at random :

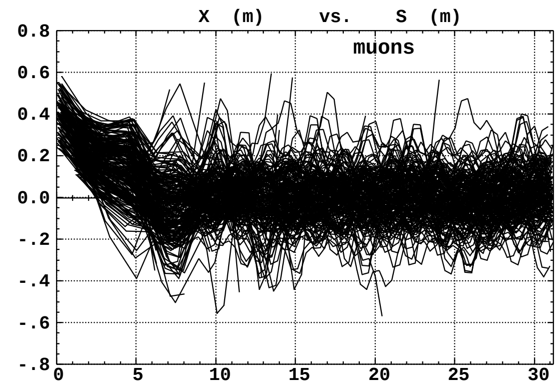
- $\theta = arcos(1 - 2R)$, R **random uniform in** $[0, 1]$
- $\phi = 2\pi R$, R **random uniform**
- **flight distance :** $s = -p_\pi/\eta \ln R$

EXAMPLE (2005+) – Neutrino Factory design studies

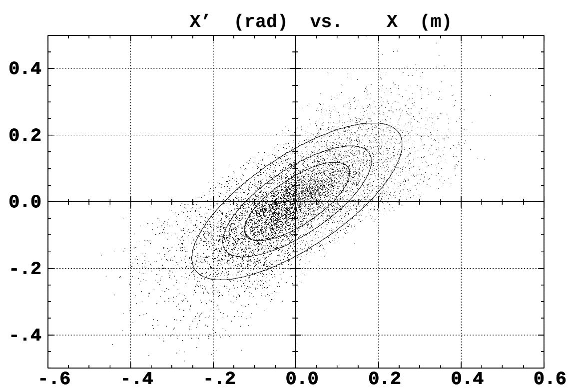
- **Objective : optimize transmission efficiency of a FODO pion collection channel, $\pi \rightarrow \mu + \nu$.**

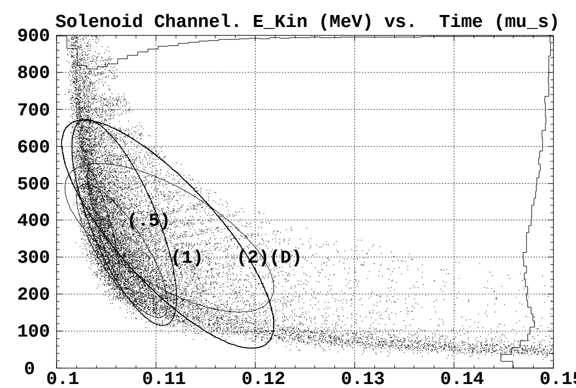
  This used the 'FIT' keyword, constraint is : maximize transmission through phase-space ellipses with given surface, at the downstream end of the line.





Sample rays in the AG Channel. Left : outermost pions. Right : decay muons.





Left : x-x' phase space at line end, $4 \cdot 10^4$ initial pions from MARS distribution.

Right : Time-energy.

9 THE FITTING PROCEDURE

Two methods installed, 1985, 2007.

An indispensable tool for
- preliminary adjustments (orbit, tunes ...)
- optimisations (higher order dynamics as DA, transmission efficiency ...)

FIT CONSTRAINTS :

Trajectory coordinates, at any location

A number of quantities deduced from trajectory coordinates, e.g. :
- first and higher order transport coefficients
- beam's α, β, emittances
- particle transmission efficiency,
- Spin coordinates
- etc.

In the case of periodic structures :
- closed orbits
- tunes, chromaticites, anharmonicities
- Spin closed orbit
- etc.

FIT VARIABLES : any data

Zgoubi input data file, EMMA :

```
'MARKER' RingInj BegRing                          start of ring. Injection point
'MULTIPOL' QD                                     start of first cell
0
7.56987 5.3 0. -2.493246 0 0 0 0 0 0 0 0
0. 0. 1.00 1.00 1.00 1.00 1.00 1. 1. 1. 1.
4 .1455 2.2670 -.6395 1.1558 0. 0. 0.
0. 0. 1.00 1.00 1.00 1.00 1.00 1. 1. 1. 1.
4 .1455 2.2670 -.6395 1.1558 0. 0. 0.
0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
0.1
2 0. 3.404834122312866 0.
'MARKER' BPM2 off                                 BPM location
'DRIFT' sd
5.00
'MULTIPOL' QF
0
5.87824 3.7 0. 2.477081 0 0 0 0 0 0 0 0
0. 0. 1.00 1.00 1.00 1.00 1.00 1. 1. 1. 1.
4 .1455 2.2670 -.6395 1.1558 0. 0. 0.
0. 0. 1.00 1.00 1.00 1.00 1.00 1. 1. 1. 1.
4 .1455 2.2670 -.6395 1.1558 0. 0. 0.
0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
0.1
2 0. 0.7513707181808552 0.
'DRIFT' ld
8.
'CAVITE'                                          accelerating cavity
7
0.736669 1.3552e9                                 Orbit length, RF frequency
70e3 0.                                           Voltage, relative phase
'MARKER' BPM1 off                                 BPM location
'CHANGREF'                                        cell orientation - wrt. next one
0. 0. -8.571428571429                             end of first cell
'REBELOTE'                                        multiturn tracking
150 0.2 99
'END'
```

CONCLUSION : THE TOTAL SIMULATION

• Given what we have seen, one may well imagine the following simulation, all integrated - one single Zgoubi run :

A high energy polarized muon FFAG decay ring (à la “MuSTORM”)

• This is fully operational. One will get :

- pion tracks and muon collection upon $\pi \longrightarrow \mu + \nu$ decay / ’MCDESINT’
- muons tracks over their few-100 turns lifetime around the ring
- evolution of neutrino beam flux with time upon $\mu \longrightarrow \nu + e$ decay
- radiative pollution by the decay electrons - they are tracked as well, if requested
- muon beam polarization and its evolution with time / ’SPNTRK’
- radiative spin diffusion / add ’SRLOSS’
- and, why not, beam diagnostics using SR !

Estimated CPU time : 0.05 second/turn * 100 turns $\approx$ 5 seconds.

BACKUP SLIDES