A NEW CLASS OF RADIAL-SECTOR CYCLOTRONS INSPIRED BY ISOCHRONOUS FFAGS

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ISOCHRONISM

For an ion of rest mass m_0 and charge q, constant orbit frequency ω, independent of energy $\gamma m_0 c^2$ and average radius R (circumf./2π), implies that

$$B = \frac{\gamma m_0 \omega}{q} = \gamma B_c\,, \qquad R = \frac{\beta c}{\omega} \equiv \beta R_c\,, \tag{1}$$

where B denotes the average field around a closed orbit in the mid-plane, B_c the "central field" and R_c the "cyclotron radius".

The resultant positive field gradient (and average field index k) produces a defocusing contribution to the vertical betatron tune ν_z:

$$\left(\Delta\nu_z^2\right)_{isoc.} = -k = -\frac{R}{B}\frac{dB}{dR} = -\frac{\beta}{\gamma}\frac{d\gamma}{d\beta} = -\beta^2\gamma^2\,. \tag{2}$$

To compensate for this Thomas suggested an azimuthal variation:

$$B_z(r,\theta) = B(r)\left(1 + f\cos N\theta\right), \tag{3}$$

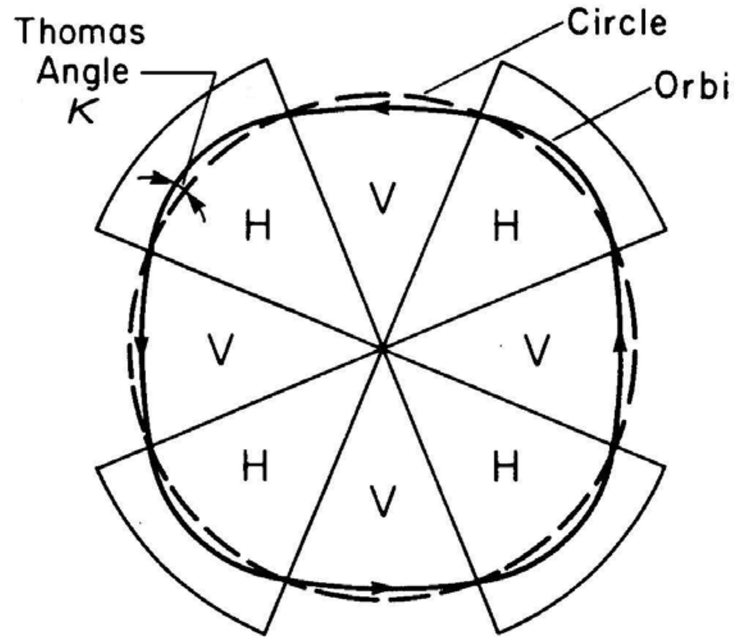
to produce a scalloped orbit and an edge focusing contribution:

$$\left(\Delta\nu_z^2\right)_{Thomas} = \tfrac{1}{2} f^2\,.$$

SECTOR-FOCUSED CYCLOTRONS

For a general variation $B_z(\theta)$ with N-fold symmetry the edge focusing is given by the "magnetic flutter" (i.e. the mean square variation in B_z:

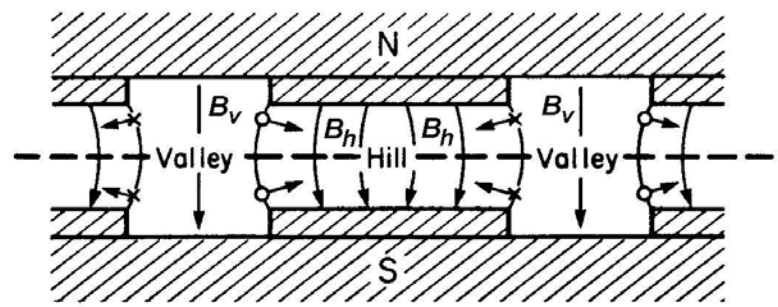
$$\boxed{F^2 \equiv \frac{\left\langle \left[B_z(\theta) - B \right]^2 \right\rangle}{B^2}.}$$



But it's hard to achieve $F^2 > 0.1$ in single-yoke "compact" cyclotrons, limiting such radial-sector machines to ~50 MeV (p).

Kerst's introduction of alternating focusing by giving the hill edges a spiral angle ε further enhances the focusing:

$$\boxed{\nu_z^2 \approx -\beta^2\gamma^2 + F^2\left(1 + \tan^2 \varepsilon\right).} \qquad (4)$$



With this powerful enhancement it has been possible to build pill-box cyclotrons to accelerate protons to 230 MeV ($\beta^2\gamma^2$ = 0.55).

SEPARATE-SECTOR CYCLOTRONS

Breaking the magnet into separate hill sectors with field-free valleys between them not only provides a more benign environment for rf and other equipment, but also increases the potential flutter. For hard-edge magnets occupying a fraction h of the orbit circumference,

$$\boxed{F^2 = \frac{1}{h} - 1.}$$

Thus the RIKEN SRC, with six 25°-wide radial sectors, is able to provide $F^2 \approx 1.4$. and accelerates light ions to 400 MeV/c ($\beta^2\gamma^2 = 1.03$). The PSI Ring Cyclotron, with eight 18°-wide spiral sectors, provides $F^2 = 1.5$ and accelerates protons to 590 MeV ($\beta^2\gamma^2 = 1.65$). Designs have been published for higher energies, ranging from 1-15 GeV.

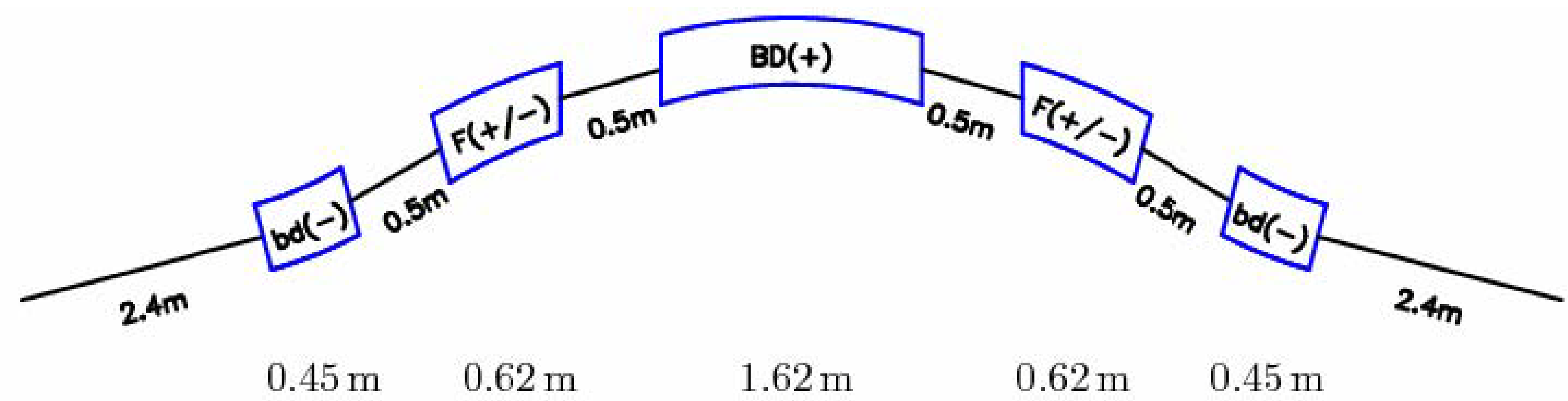
REVERSE-BEND CYCLOTRONS

- where $B_v = -B_h$, as in radial-sector FFAGs, can produce even higher flutter, and also significant AG focusing. A design with $h = 0.6$, $F^2 = 24$, (enough to counter $\beta^2\gamma^2$ at 3.7 GeV) was found to focus up to 5.9 GeV.

ISOCHRONOUS FFAGS I

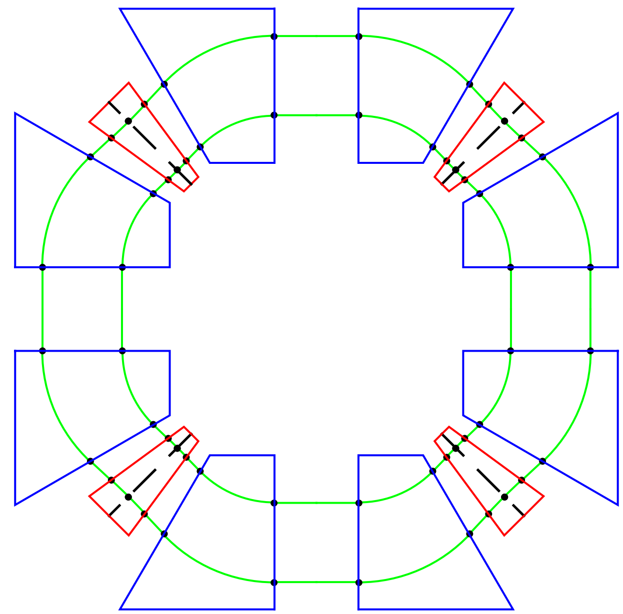
But isochronous FFAGs are capable of much higher energies!

Grahame Rees has designed several FFAGs using novel 5-magnet "pumplet" cells, in which variations in field gradient and sign enable each magnet's function to vary with radius – providing great flexibility – even allowing well-matched insertions!



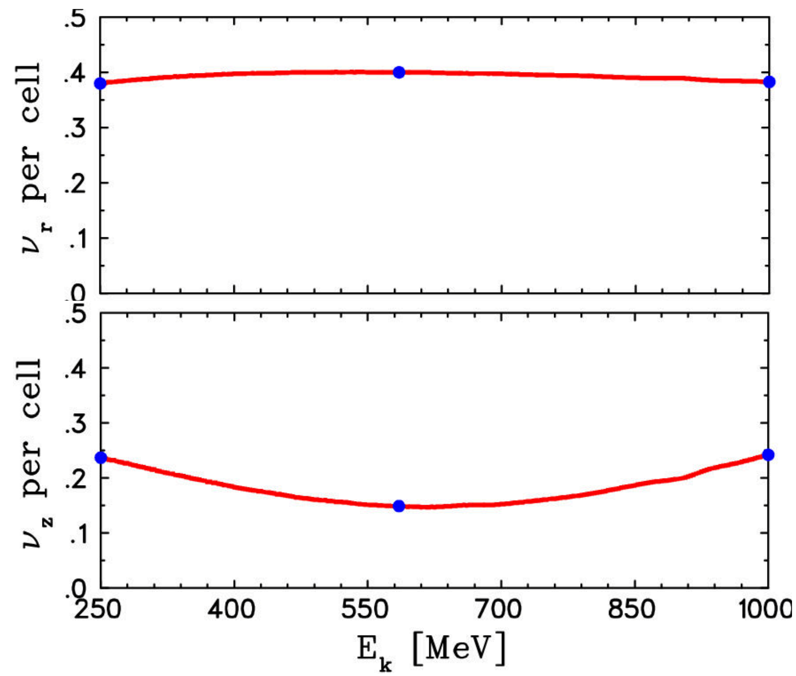
Among them was an isochronous "IFFAG" (C = 1255 m, $N = 123$, 16 turns) for 8-20 GeV muons – i.e. $5{,}900 < \beta^2\gamma^2 < 37{,}000$!

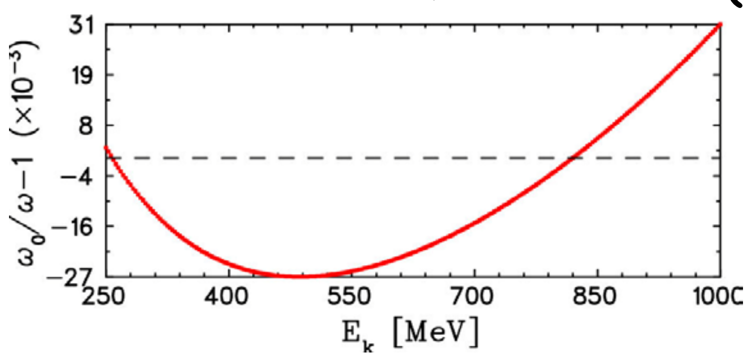
ISOCHRONOUS FFAGS II



Carol Johnstone has proposed a two-stage proton LNS-FFAG, operating at fixed frequency, for ADSR. Each stage uses a 4-cell FDF triplet lattice (right) with straight-sided (though not necessarily radial) edges and a specially determined $B(r)$ profile.

Tracking studies of the second stage (250-1000 MeV) with CYCLOPS have confirmed that the orbits are close to isochronous and the tunes (ν_z and ν_r) near constant. The CYCLOPS results (—) agree well with those from COSY (•).





DIFFERENT HILL AND VALLEY FIELD PROFILES I

Note that in all the cyclotron schemes described above the functional dependences of B_z on r and θ are assumed independent:

$$\boxed{B_Z = f(r)g(\theta),}$$

and in particular that the hill and valley fields have the same profile:

$$\boxed{B_v(r)/B_h(r) = \text{constant}.}$$

Thus in a radial-sector cyclotron there's only one free parameter – the flutter - available to control the vertical tune.

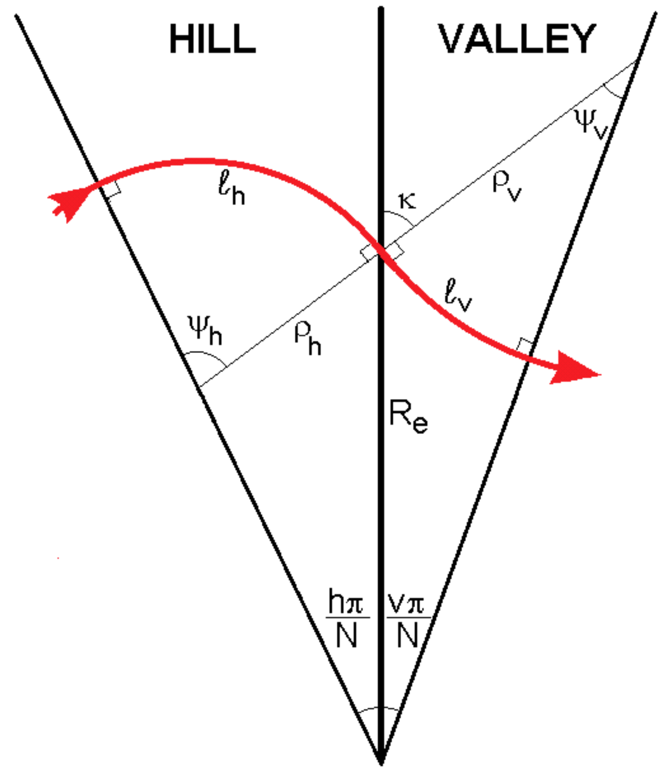
To provide more freedom of action and achieve positive vertical focusing at higher energies, we have explored a simpler possibility than in the FFAGs - allowing the radial field profiles in hills and valleys to differ – and assuming a polynomial variation with energy:

$$\boxed{B_h(\gamma) = H_0 + H_1\gamma + H_2\gamma^2 + H_3\gamma^3 + ...} \qquad (5)$$

$$\boxed{B_v(\gamma) = V_0 + V_1\gamma + V_2\gamma^2 + V_3\gamma^3 + ...} \qquad (6)$$

A COMPACT DESIGN WITH NEGATIVE VALLEY FIELDS

As a first step we consider a "compact" design with no drift spaces, negative valley fields, hard edges, and $\boldsymbol{B_h}$ and $\boldsymbol{B_v}$ each constant along equilibrium orbits. If $\ell_h = \rho_h \psi_h$ and $\ell_v = \rho_v \psi_v$ are the arc lengths within a half-cell, then to maintain isochronism,



$$\boxed{\begin{array}{l} \ell_h H_1 + \ell_v V_1 = \dfrac{\pi}{N} B_c R_c \beta \quad \text{and} \\ \ell_h H_n + \ell_v V_n = 0 \;\; (n \neq 1). \end{array}}$$

If the hill coefficients H_n are specified, the valley coefficients V_n must satisfy:

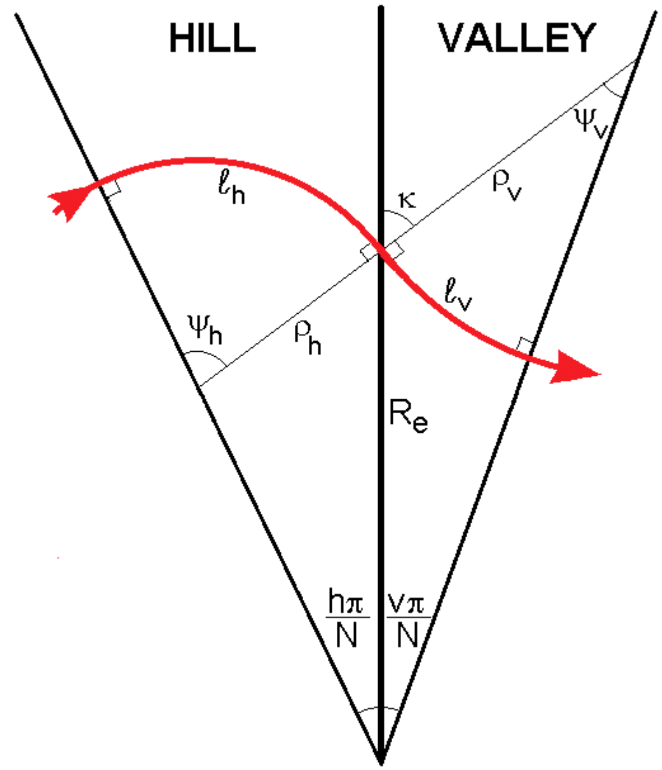
$$\boxed{\begin{array}{l} V_1 = \dfrac{\pi}{N} \dfrac{B_c R_c}{\ell_v} \beta - \dfrac{\ell_h}{\ell_v} H_1 \quad \text{and} \\ V_n = -\dfrac{\ell_h}{\ell_v} H_n \;\; (n \neq 1). \end{array}}$$

So to compute them we need the values of ℓ_h and ℓ_v.

ORBIT GEOMETRY

Computing the valley field B_v requires a knowledge of ℓ_h and ℓ_v, and therefore of the bending angles ψ_h and ψ_v and the radii of curvature – of which ρ_v itself depends on B_v!

These parameters may nevertheless be evaluated by invoking their various geometrical relationships, which after some manipulation yield a transcendental equation for ψ_h, from which the other parameters follow:



$$\boxed{\psi_h + (\psi_h - \pi/N)\frac{\sin\psi_h \sin[(1-h)\pi/N]}{\sin(\psi_h - \pi/N)\sin(h\pi/N)} = \frac{B_c\beta\gamma}{B_h(\gamma)}.}$$

This must be solved numerically, but a good starting point is to make the approximation $R_e = \beta R_c$, giving:

$$\boxed{\psi_{h0} = \arcsin\left(\frac{B_h(\gamma)}{\gamma B_c}\sin\left(h\pi/N\right)\right).}$$

BETATRON TUNES

To calculate the tunes we take a lumped-element approach (validated by tracking with CYCLOPS in previous studies):

$$\boxed{\cos(2\pi\nu / N) = \tfrac{1}{2}\mathrm{Tr}(M_e M_v M_e M_h),}$$

where M_e is the standard 2×2 matrix for a thin lens, while for vertical motion, M_v and M_h are those for focusing and defocusing sector magnets respectively. For M_e we need the focal power g of the edge crossing, given by:

$$\boxed{g = \frac{B_h - B_v}{B_c R_c \beta\gamma} \tan\left(\psi_h - \frac{h\pi}{N}\right).}$$

For M_v and M_h we need the phase advances $\phi_{h,v} = \ell_{h,v} \sqrt{K_{h,v}}$, where:

$$\boxed{K_h = \frac{dB_h / dr}{B_c R_c \beta\gamma} = \frac{\gamma^2}{B_c R_c^2}\left(H_1 + 2H_2\gamma + 3H_3\gamma^2 + \ldots\right),}$$

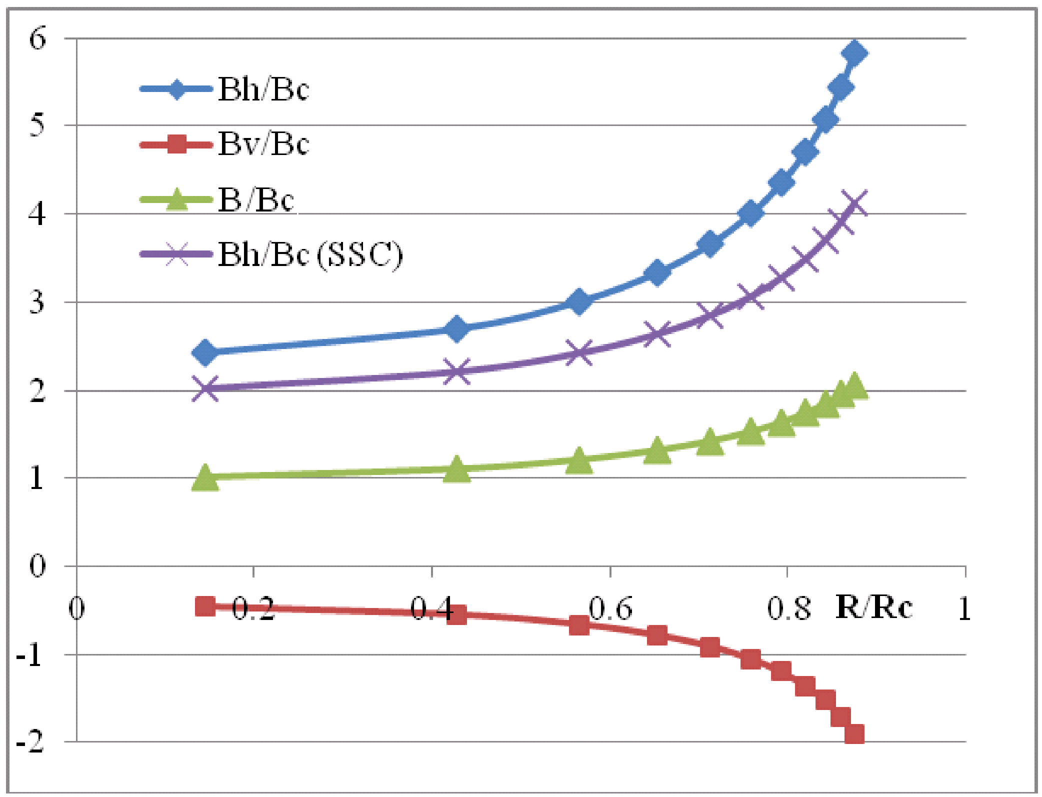
$$\boxed{K_v = \frac{dB_v / dr}{B_c R_c \beta\gamma} = \frac{\gamma^2}{B_c R_c^2}\left(V_1 + 2V_2\gamma + 3V_3\gamma^2 + \ldots\right).}$$

For horizontal motion $\phi^*_{h,v} = \ell_{h,v} \sqrt{K^*_{h,v}}$, where $K^*_{h,v} = (1/\rho_{h,v})^2 \pm K_{h,v}$.

CASES STUDIED

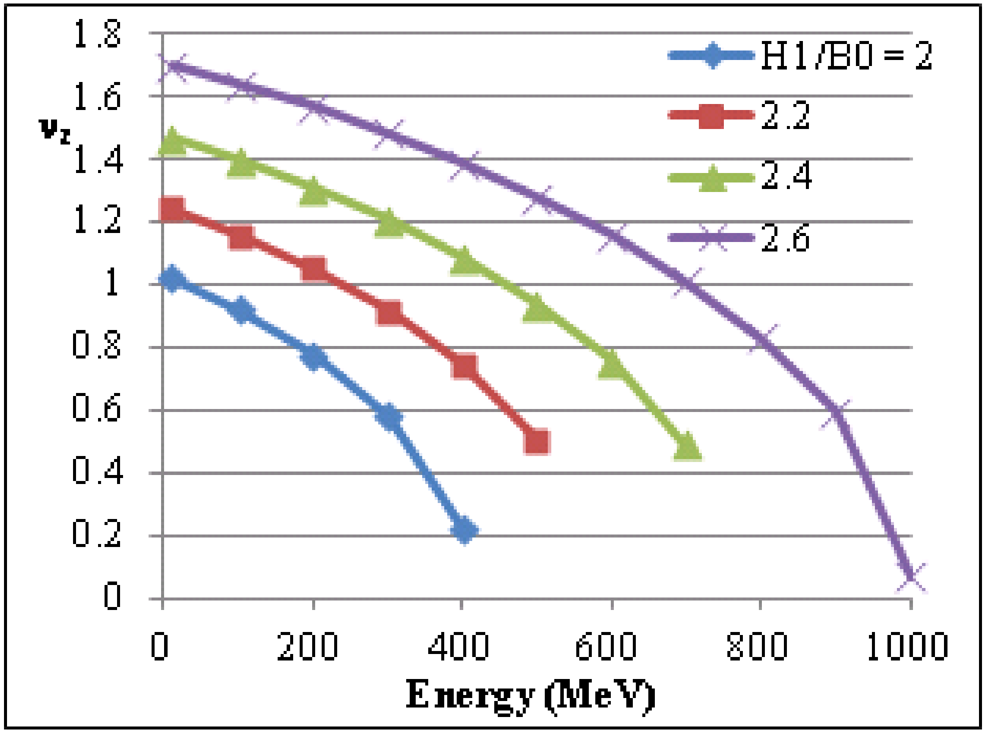
A number of cases were studied to investigate the effect on the tunes of simply adding a γ^2 component to the conventional γ-variation (i.e. $H_n = 0 = H_{n>2}$). Their dependence on the hill fraction h and the number of sectors N were also investigated, though most runs were made for $h = 0.5$ and $N = 8$.

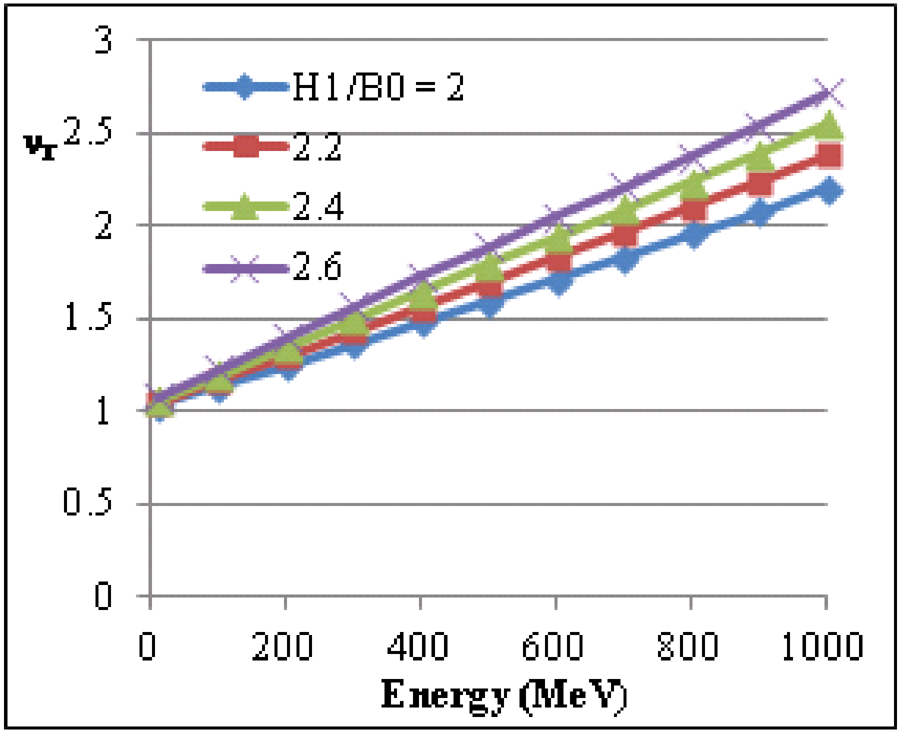
The figure displays an example of such field profiles for $H_1 = 2B_c$, $H_2 = 0.4B_c$ - one of the more promising cases studied, for which ν_z varies little from 10-1000 MeV.



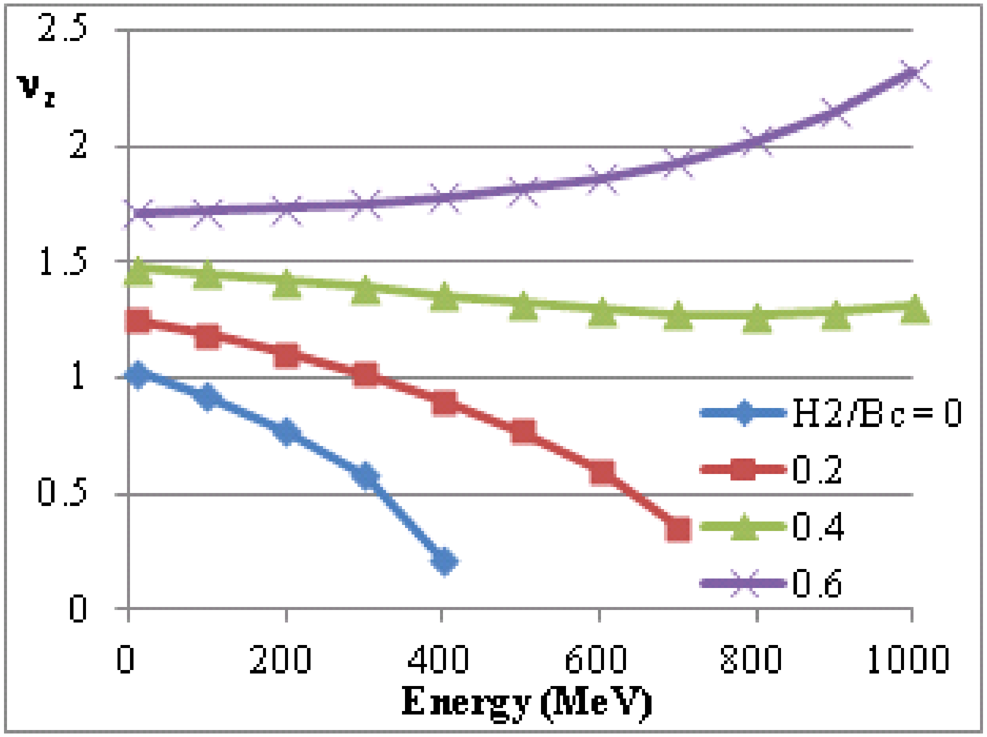
The comparison curve for a separate-sector cyclotron (SSC) with the same $h = 0.5$ shows that for $H_2 = 0.2H_1$ the B_h required would be 20-40% higher.

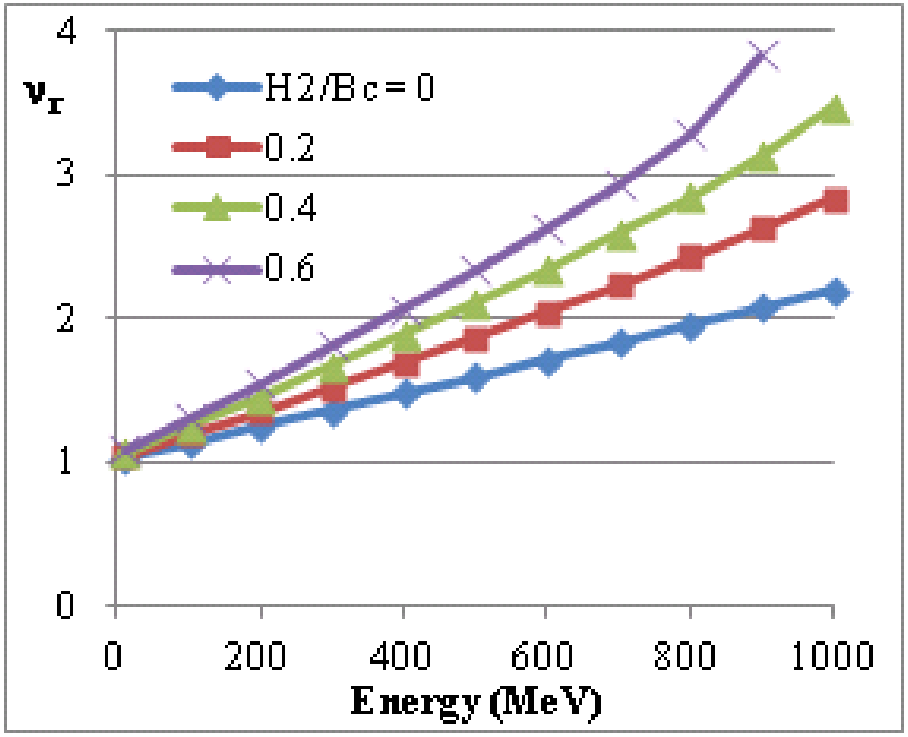
REFERENCE CASE: NO γ^2 COMPONENT
(SEPARATE-SECTOR & NEGATIVE-BEND CYCLOTRONS)
($H_2 = 0$, $h = 0.5$, $N = 8$)



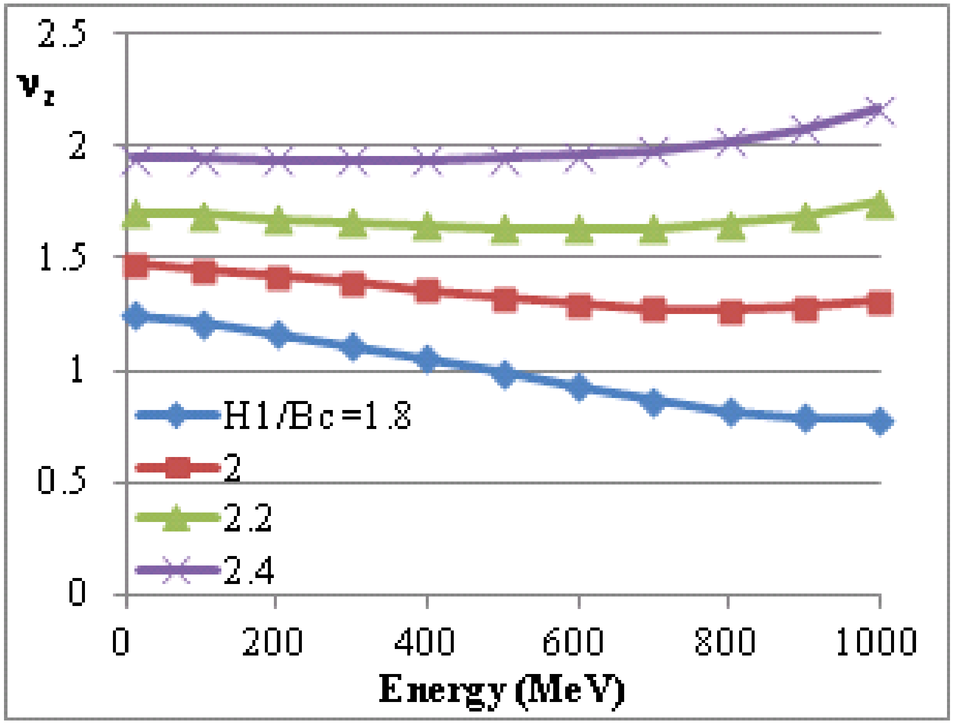


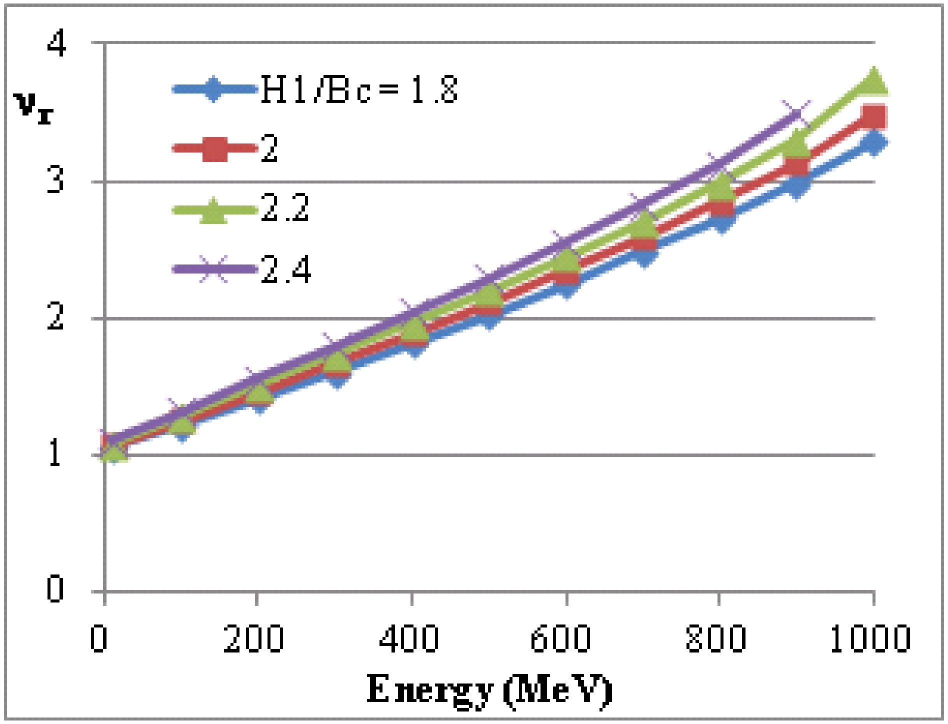
EFFECT OF ADDING A γ^2 COMPONENT
$(H_1/B_c = 2, h = 0.5, N = 8)$





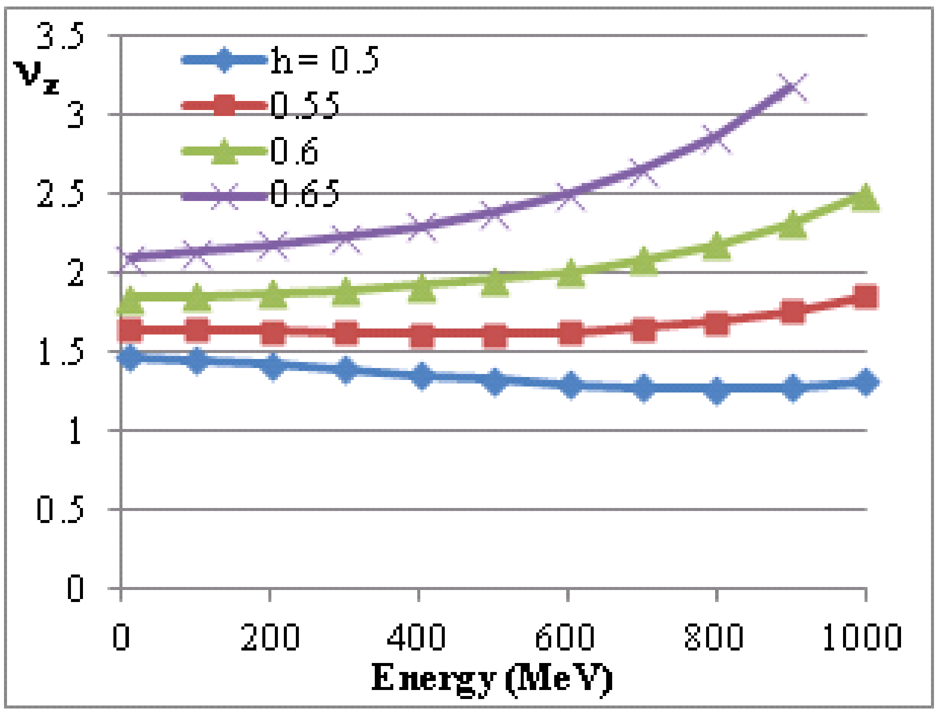
EFFECT OF VARYING H_1 FOR FIXED H_2
(H_2 = 0.4, h = 0.5, N = 8)

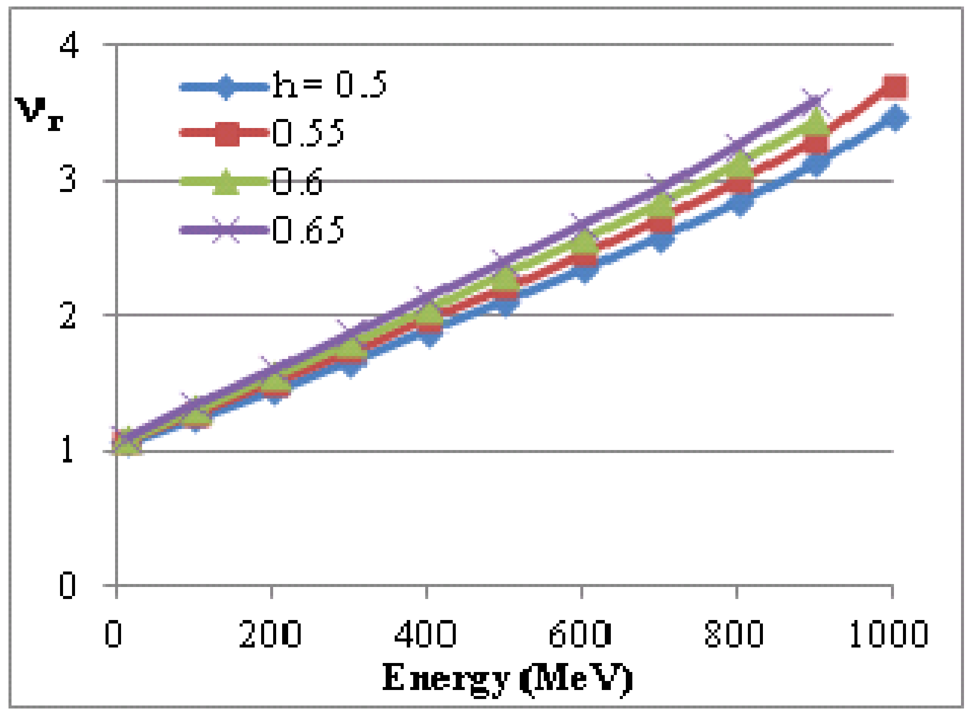




EFFECT OF VARYING h FOR FIXED H_1, H_2

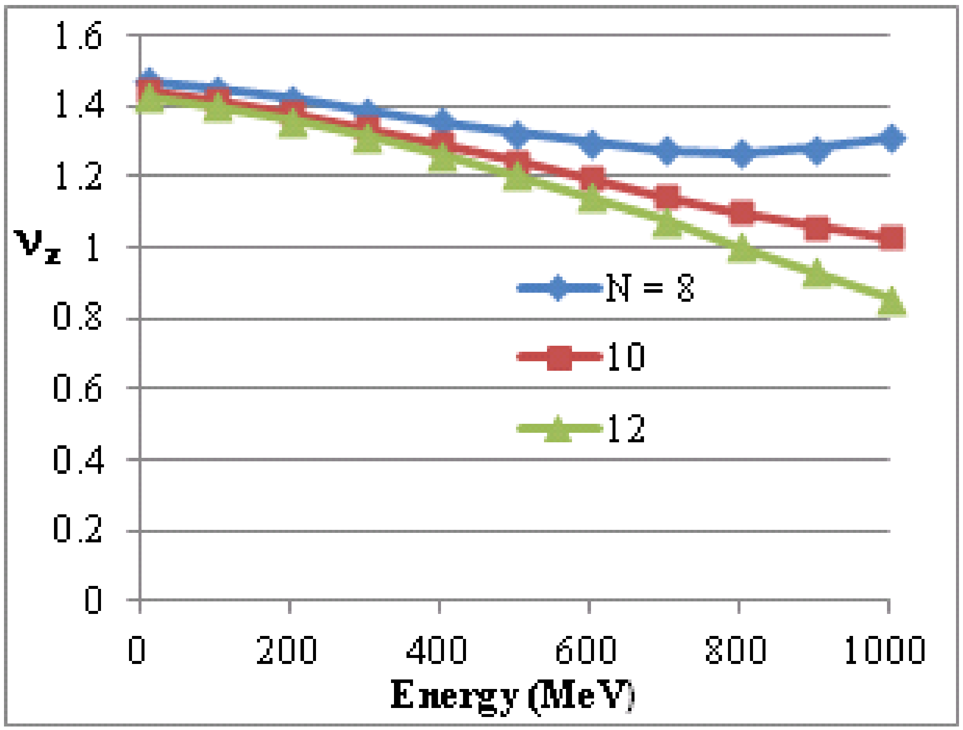
$(H_1/B_c = 2, H_2/B_c = 0.4, N = 8)$

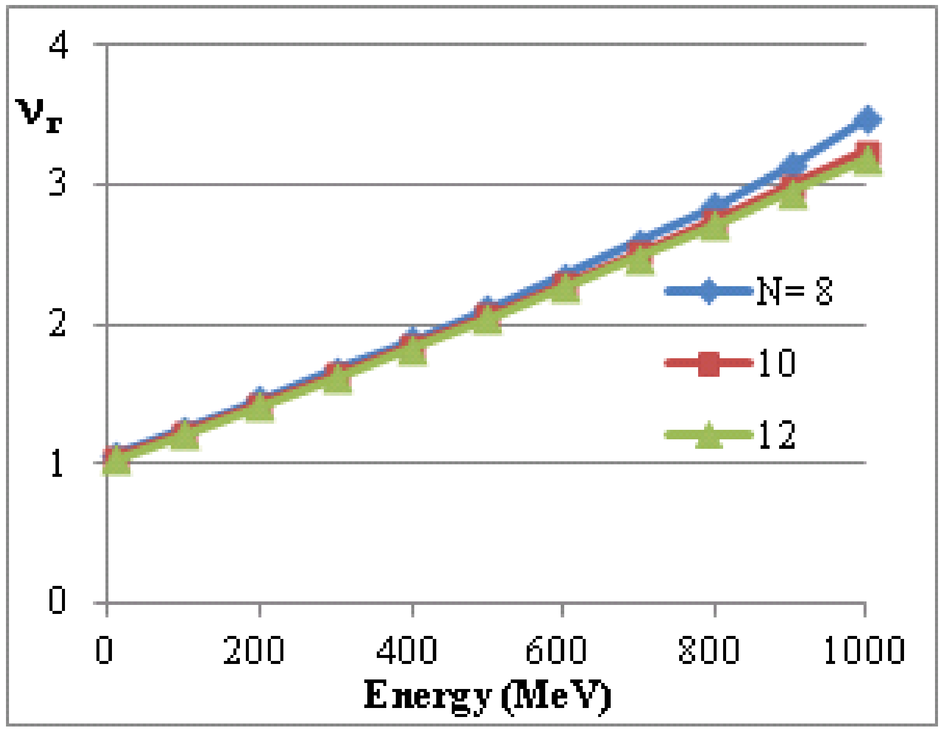




EFFECT OF VARYING N FOR FIXED H_1, H_2

(H_1/B_c = 2, H_2/B_c = 0.4, h = 0.5)





SUMMARY & CONCLUSIONS

- A study has begun of using different radial field profiles in hills and valleys (while maintaining isochronism) to obtain increased flutter and more strongly alternating gradients – and hence increased vertical focusing - in radial-sector cyclotrons.
- As a first step, adding a γ^2 component to the hill fields in a "compact" design (i.e. no field-free regions) - and subtracting a compensating γ^2 component from the valley fields - has been shown to be a possible way of providing radial-sector cyclotrons with sufficient vertical focusing to reach at least 1 GeV.
- The practicality of such a design has not been taken into account, particularly with regard to finding suitable locations for the rf accelerating cavities and injection and extraction systems. Field-free drift spaces would remove this difficulty and a beam optics study of such an arrangement has begun.